Topographic effects in a stratified layer at the top of the core

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Rémy Monville, David Cébron, Dominique Jault – 2021

ISTerre, Université Grenoble Alpes, CNRS

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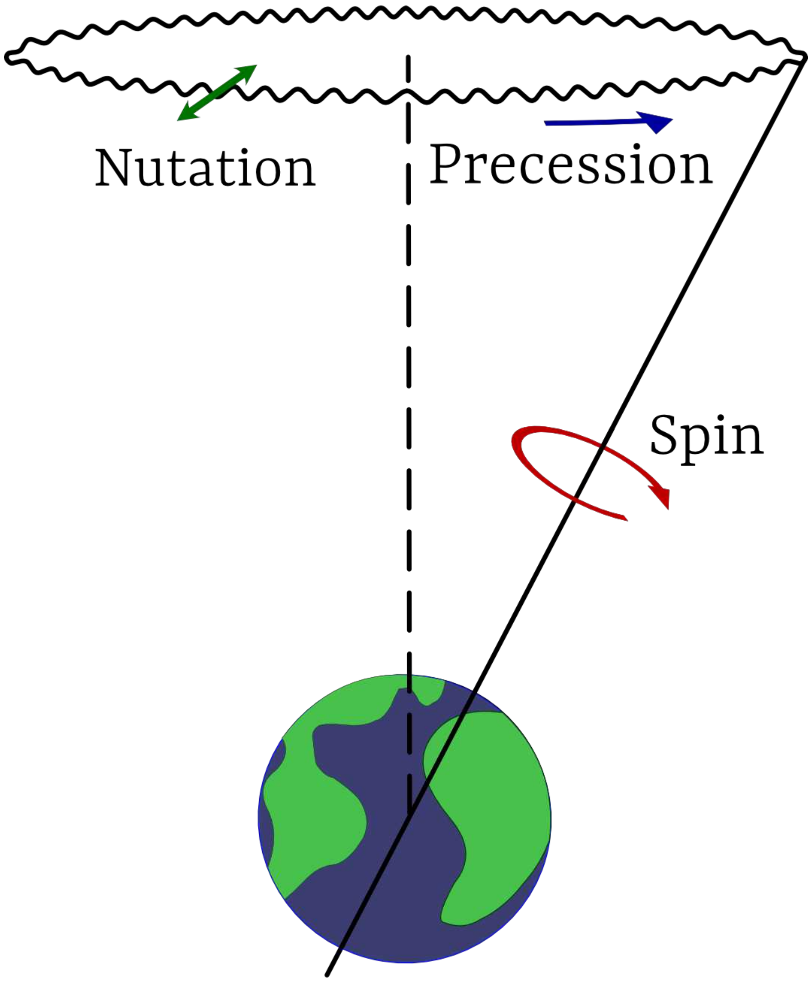
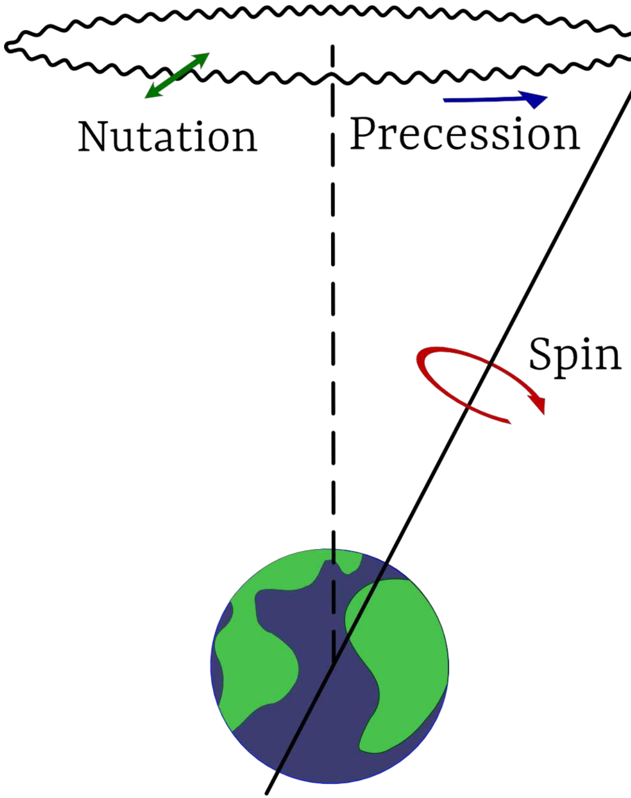
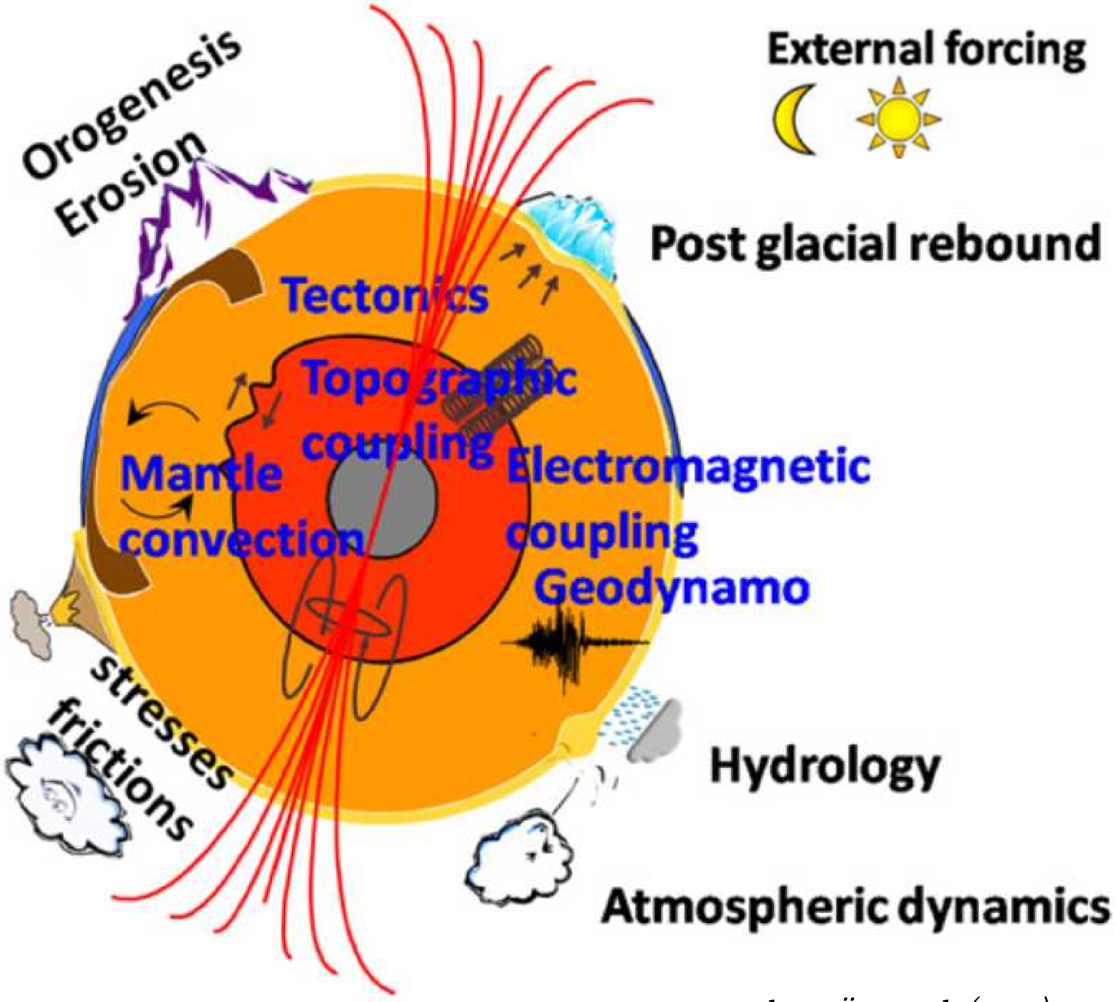
Variations of the Earth's rotation



Variations of the Earth's rotation



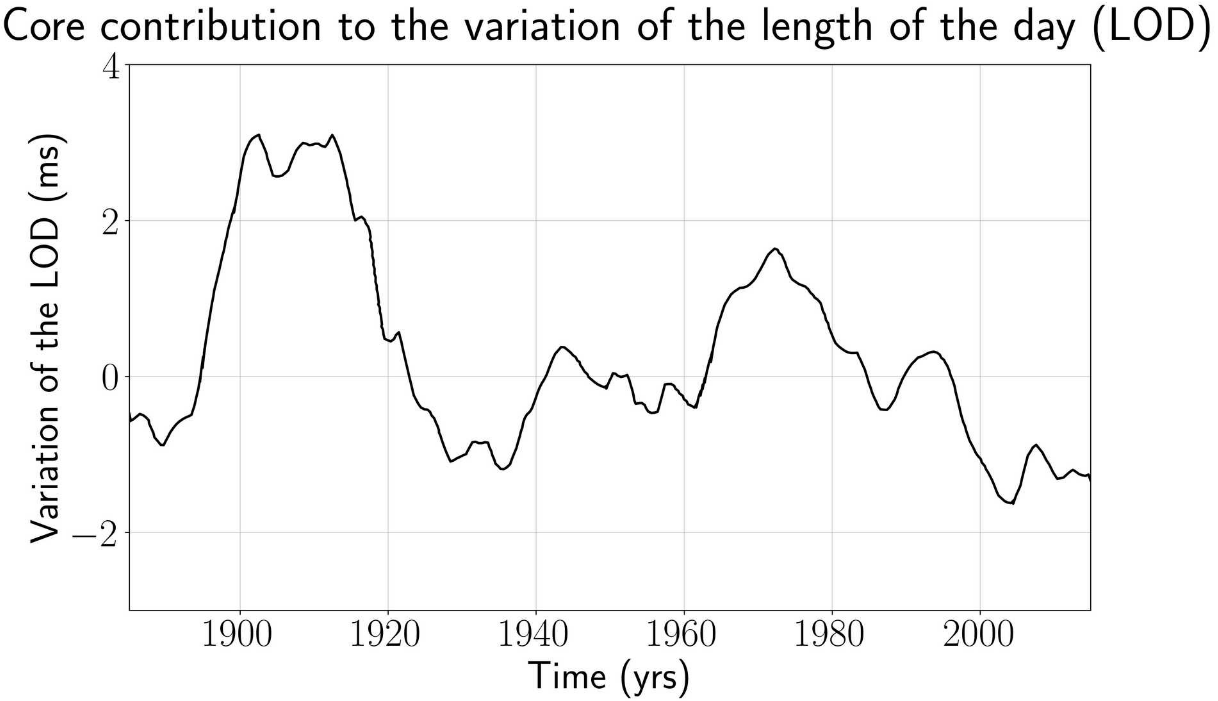


Karatekin, Ö. et al. (2011)

Exchanges of angular momentum

Core coupling:

- **Electromagnetic**
- **Viscous**
- **Gravitational**
- **Pressure torque on small scale topography**



Gillet, et al. (2019).

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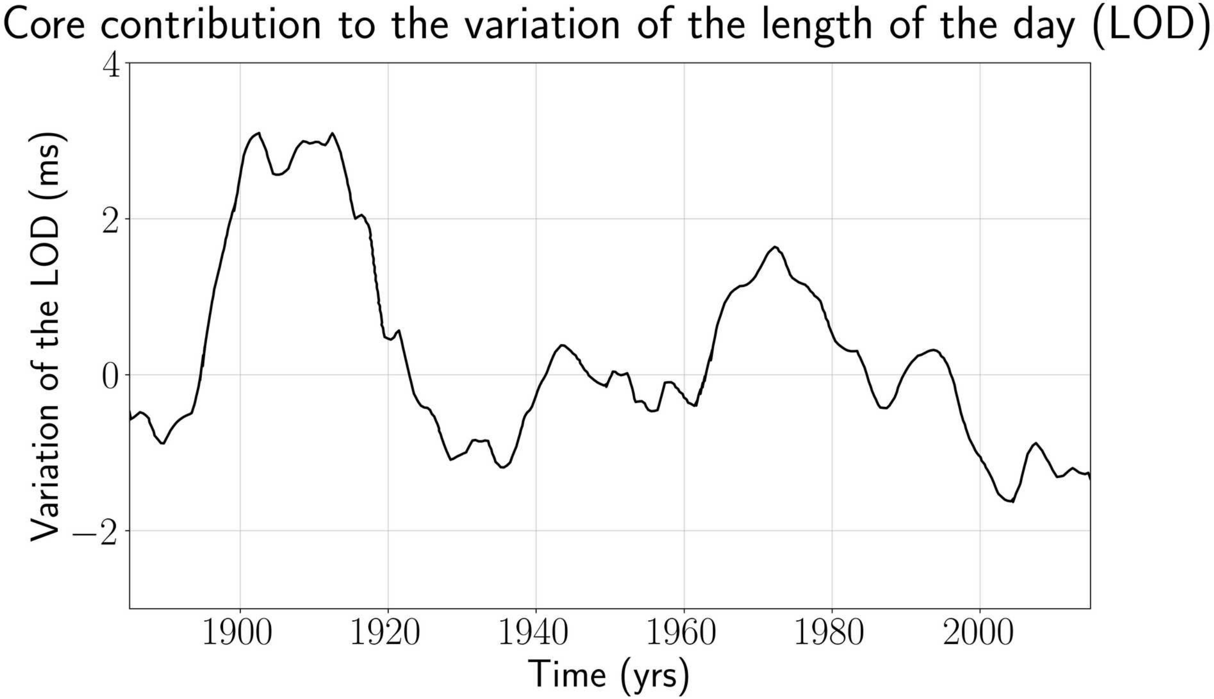

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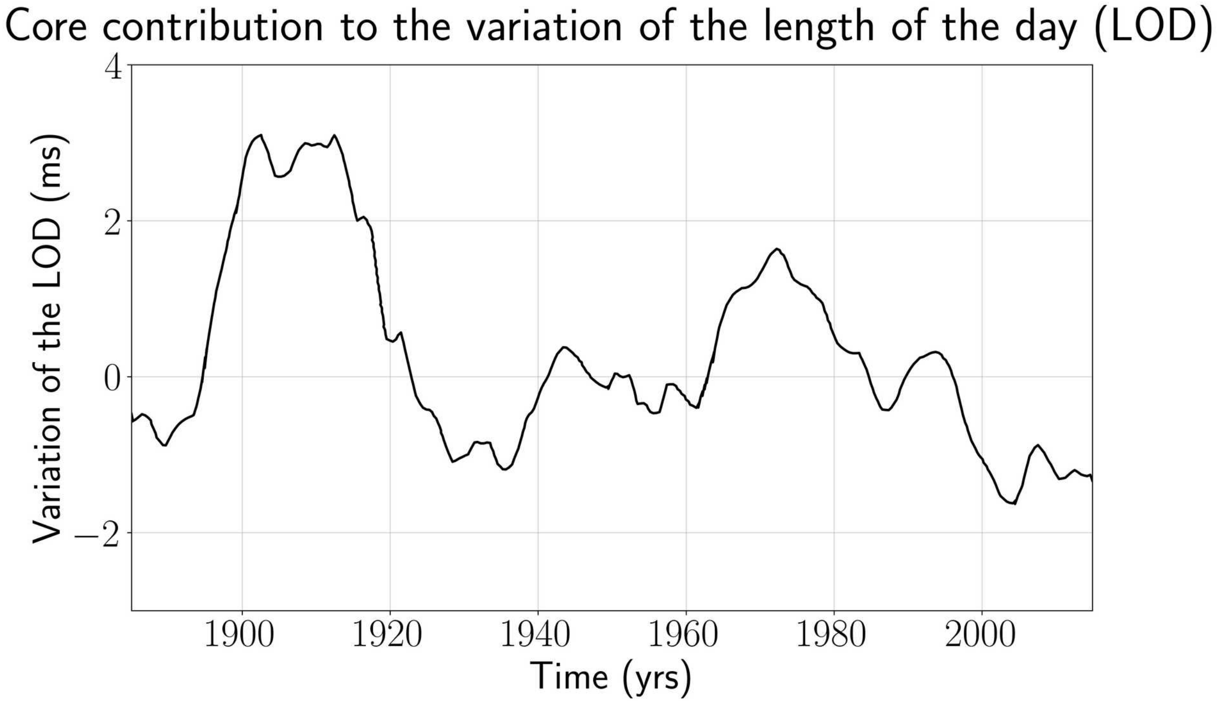


Gillet, et al. (2019).

Motivations

Can the **small scale** topographic coupling explain:

- The decadal changes in the **Length-of-Day** *(Glane and Buffett 2018, Jault 2020)* ?
- The out of phase component of the retrograde annual **nutation** of the Earth's rotation axis *(Buffett 2010)* ?

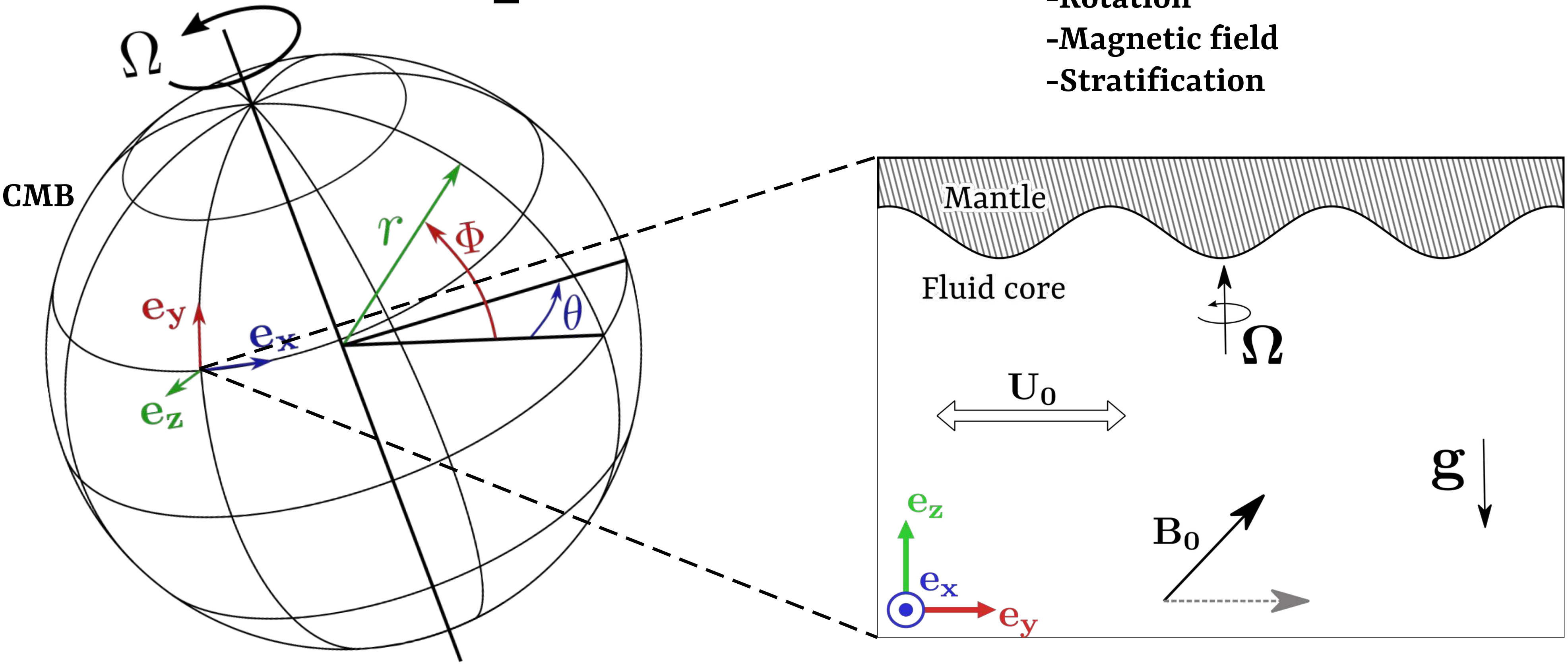
How well can a **local perturbative model** help us to understand these measurements, and what are its limitations ?

Geometry of the problem

Spherical shell → Cartesian frame

Key effects:

- **-Rotation**
- **-Magnetic field**
- **-Stratification**



Geometry of the problem

Basic State:

- **Velocity:**

$$\mathbf{U_0} = u_0 \, \Re_e \left(e^{I\omega t} \right) \mathbf{e_x}$$

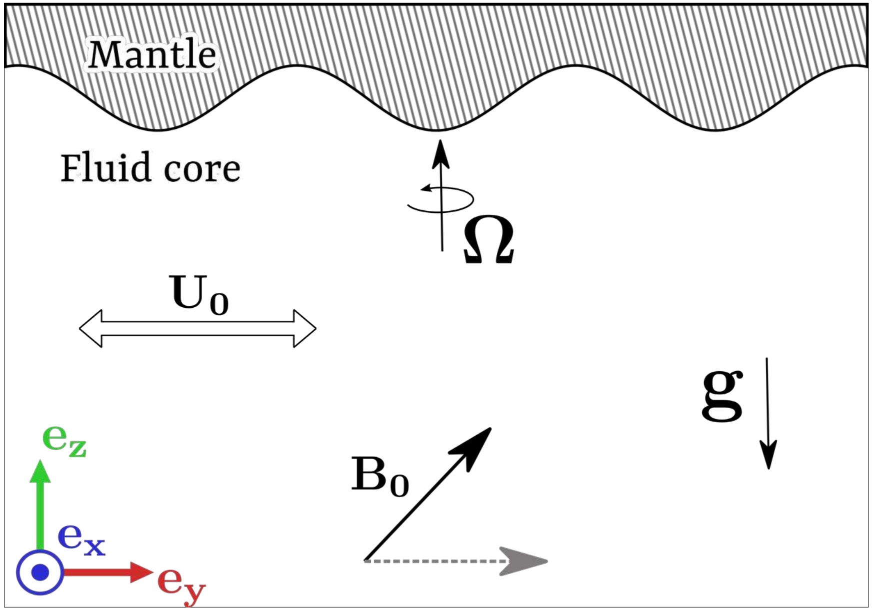
or →

- **Magnetic field:**

$$\mathbf{B_0} = b_0 \left(\cos(\Phi) \mathbf{e_y} - 2 \sin(\Phi) \mathbf{e_z} \right)$$

- **Density:**

$$\rho = \rho_r (1 - \alpha z)$$



Equations of motion

Navier-Stokes

$$\rho_r D_t \mathbf{U} = -\rho_r \chi \mathbf{e_z} \times \mathbf{U} - \nabla p + \mu \nabla^2 \mathbf{U} + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B},$$

Mass conservation → $$\partial_t \rho + (\mathbf{U} \cdot \nabla)\rho = 0,$$

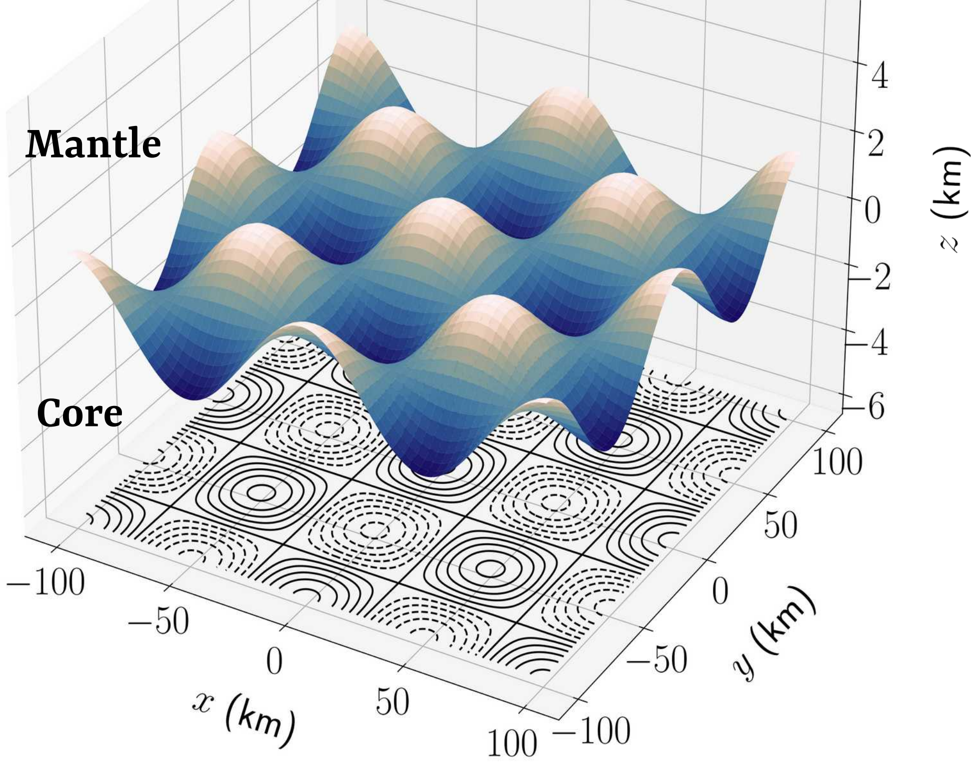
Magnetic field (Induction) → $$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{U} \times \mathbf{B}),$$

Incompressible fluid → $$\nabla \cdot \mathbf{U} = 0,$$

Conservation of magnetic flux → $$\nabla \cdot \mathbf{B} = 0,$$

Magneto-hydro-dynamic equations (MHD), in Boussinesq approximation

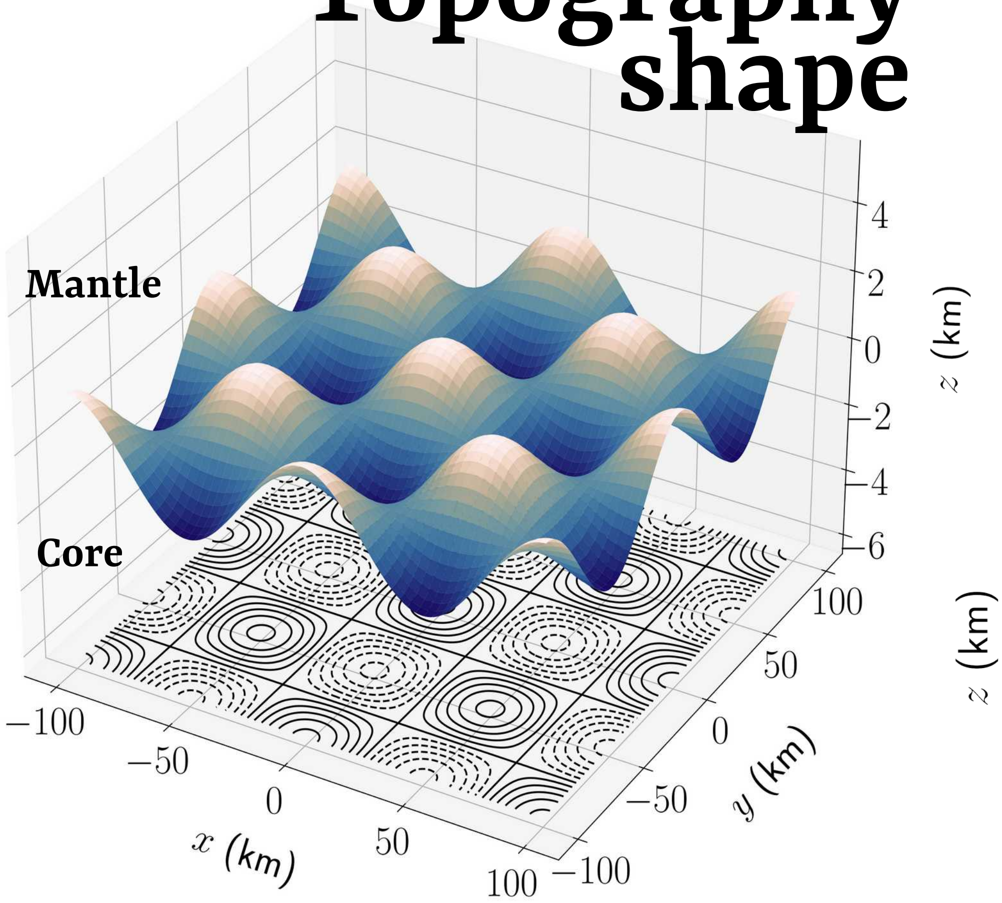
Topography shape



$$h(x, y) = \epsilon_t \sum_{j=0}^{n} \Re \left(\exp \left(i\mathbf{k_j} \cdot \mathbf{r} \right) \right) / n,$$

$\epsilon_t \ll 1$ is the topography height divided by a typical length scale

Topography shape



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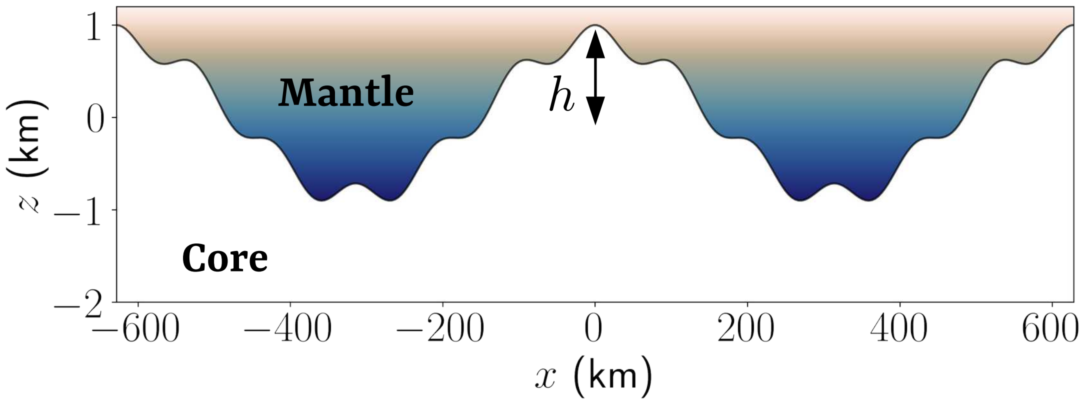


Methods

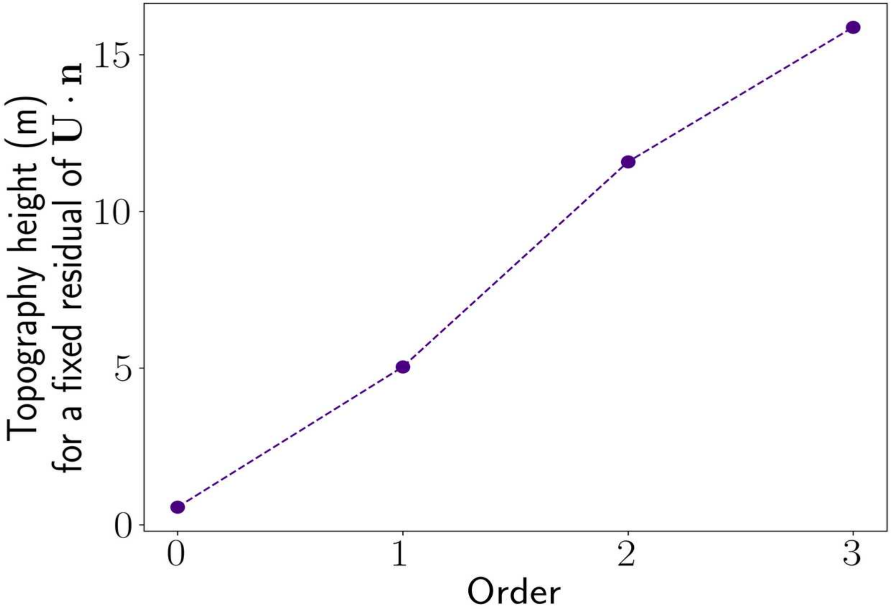
Solving equations with with a weakly non linear perturbation approach

Limited by a small parameter
→ Topography height

Glane and Buffett (2018) : ~30-50m

Buffett (2010) : ~100m

Higher orders of perturbation



Quasi linear variation when the serie is convergent

Methods

2 steps

Derive the equations

Solve systems

Methods

2 steps

Derive the equations → *Sympy : symbolic mathematics*

Solve systems → *mpmath : arithmetic with arbitrary precision*

Methods

2 steps

Derive the equations

Solve systems

Sympy : symbolic mathematics

python™

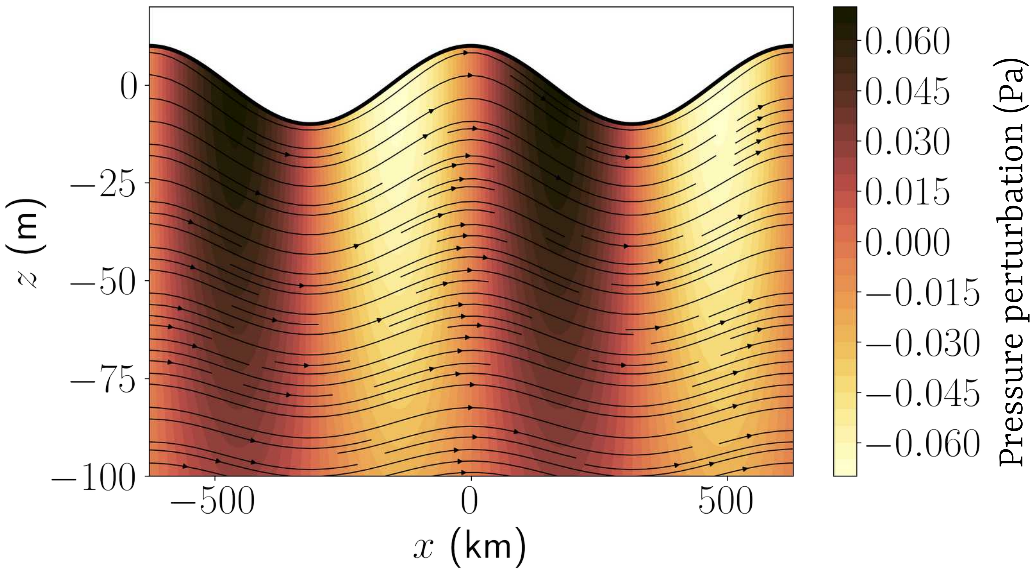
mpmath

$\to \infty$

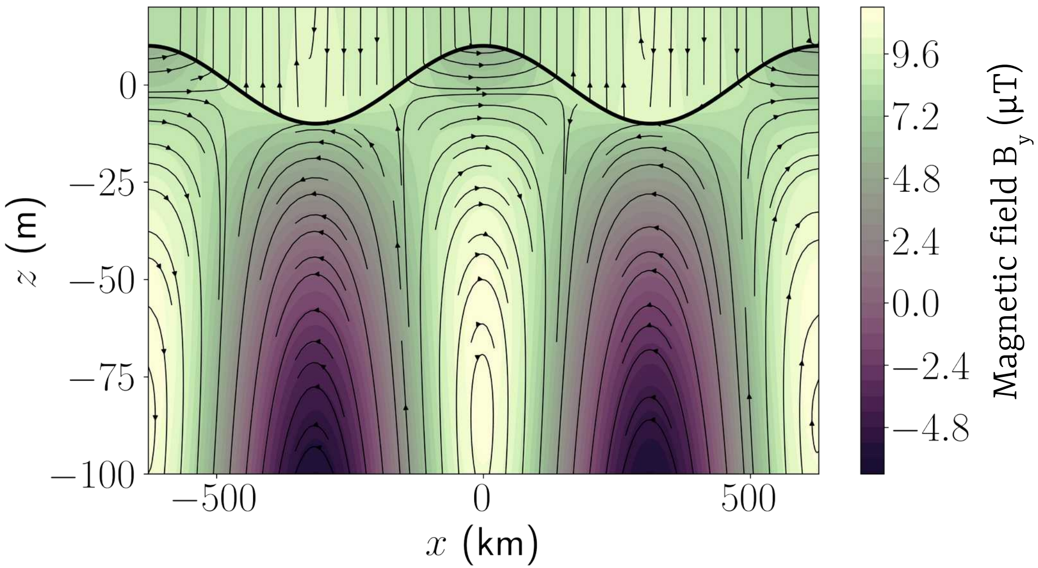
mpmath : arithmetic with arbitrary precision

My code : **ToCCo**

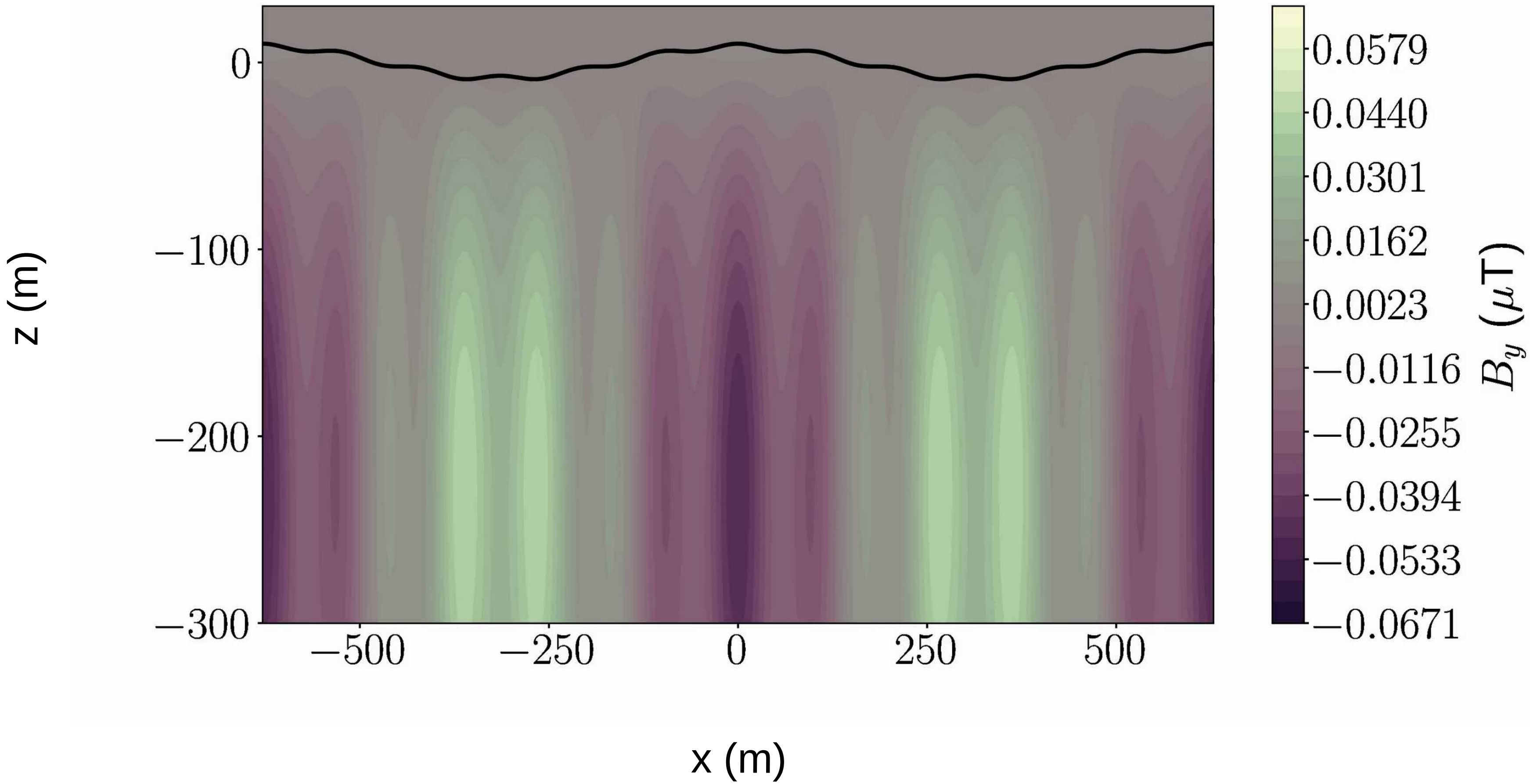
Results



Streamlines = velocity



Streamlines = Current density

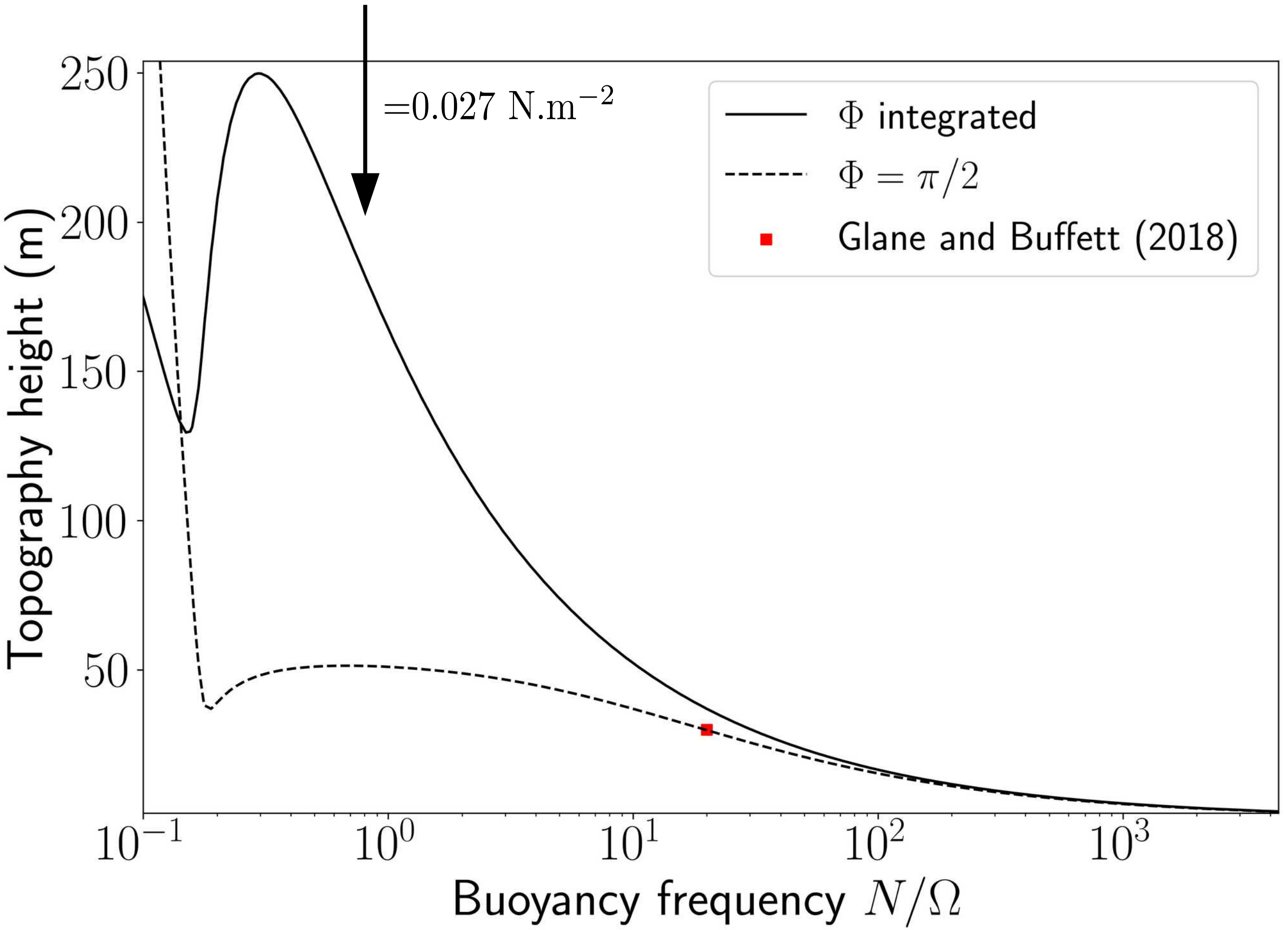


0
−100
−200
−300
−500
−250
0
250
500
0.0579
0.0440
0.0301
0.0162
0.0023
−0.0116
−0.0255
−0.0394
−0.0533
−0.0671
B_y (μT)
z (m)
x (m)

What is the value for (h,N) required to explain the observed variation of the length of the day ?

- Steady and uniform flow

- Insulating mantle



integrated = integration with latitude, taking into account the variation of $\mathbf{\Omega}$ and $\mathbf{B_0}$

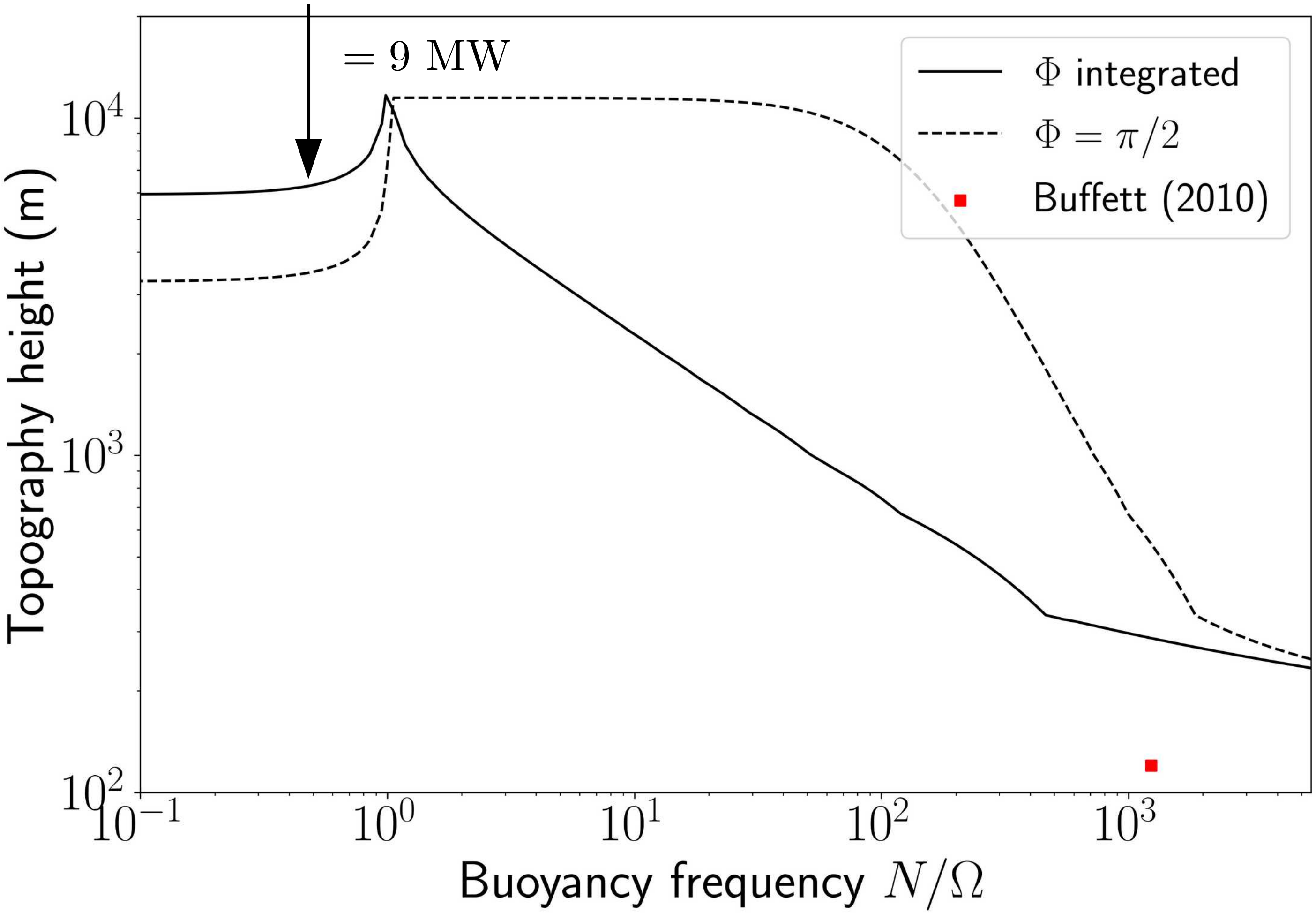
What is the value for (h,N) required to explain the observed dissipative coupling ?

- Oscillating flow with diurnal period

- Conducting mantle
  → electrical conductivity ratio : $\frac{\sigma_{core}}{\sigma_{mantle}} = 500$

- At the pole : $B_0 = 0.5$ mT

integrated = integration with latitude, taking into account the variation of $\mathbf{\Omega}$ and $\mathbf{B_0}$



Comparing at the same parameters

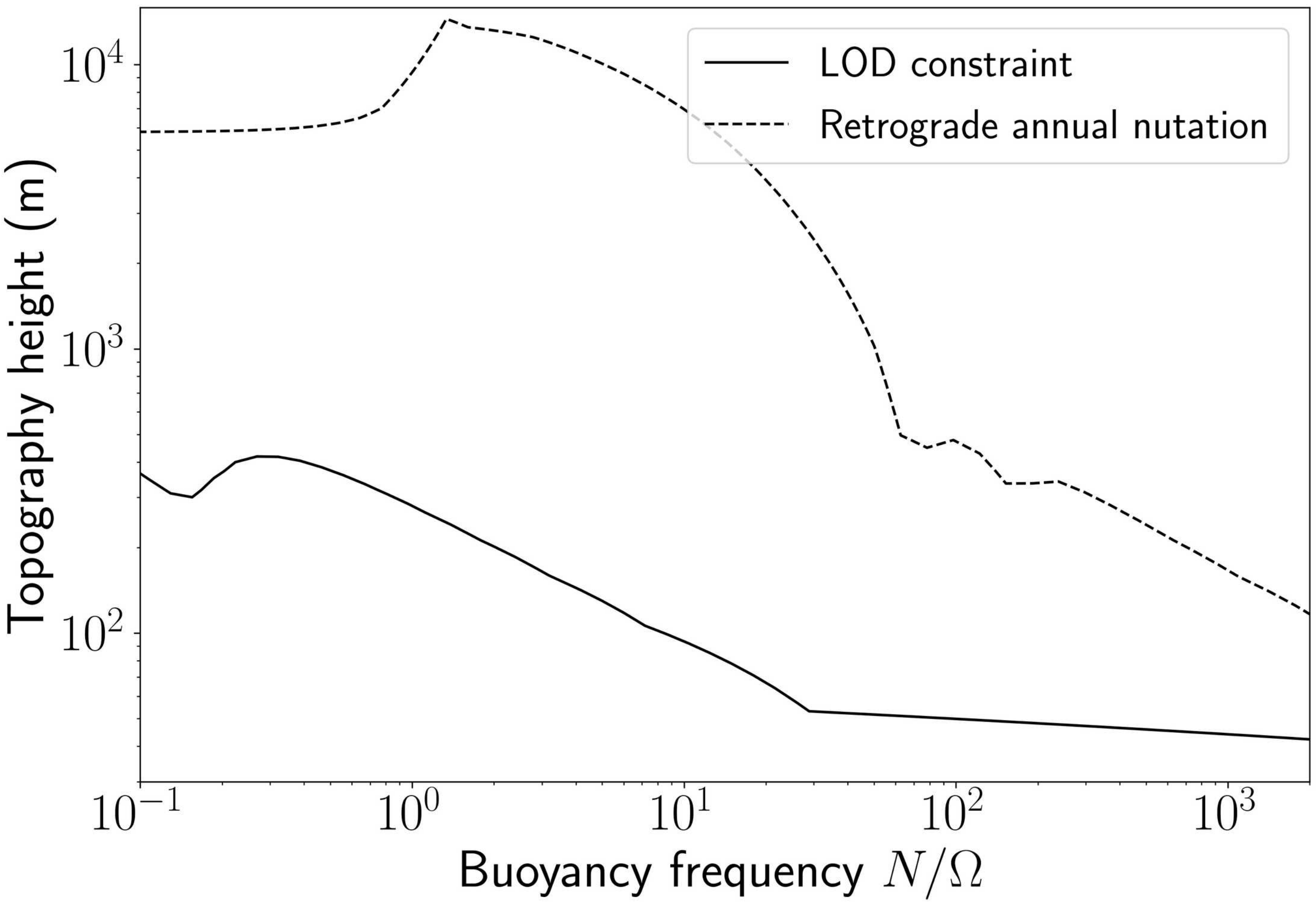
-Integrated with latitude

-Conducting mantle

→ electrical conductivity

ratio : $\frac{\sigma_{core}}{\sigma_{mantle}} = 500$

- At the pole : $B_0 = 0.65$ mT



Conclusion

- We can, with our model, explore a wide panel of parameters in a consistent manner
- developed at a higher order of perturbation
- Our model, in its simplest form, does not explain simultaneously the two sets of data (LOD, nutation)

and Perspectives

- Couple our results with Earth model of rotation
- Study the convergence radius of the perturbative model and constrain its limit of applicability.

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Equations of motion

Inertia — Coriolis force (with Beta-plane approximation) — Reduced pressure gradient — Inviscid fluid — Buoyancy — Lorentz force

$$\rho_r D_t \mathbf{U} = -\rho_r \chi \mathbf{e_z} \times \mathbf{U} - \nabla p + \mu \nabla^2 \mathbf{U} + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B},$$

$$\partial_t \rho + (\mathbf{U} \cdot \nabla) \rho = 0,$$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{U} \times \mathbf{B}),$$

Joule Dissipation — Coupling term

$$\nabla \cdot \mathbf{U} = 0,$$

Magneto-hydro-dynamic equations (MHD)