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# Topographic effects in planetary magneto-hydrodynamic flows

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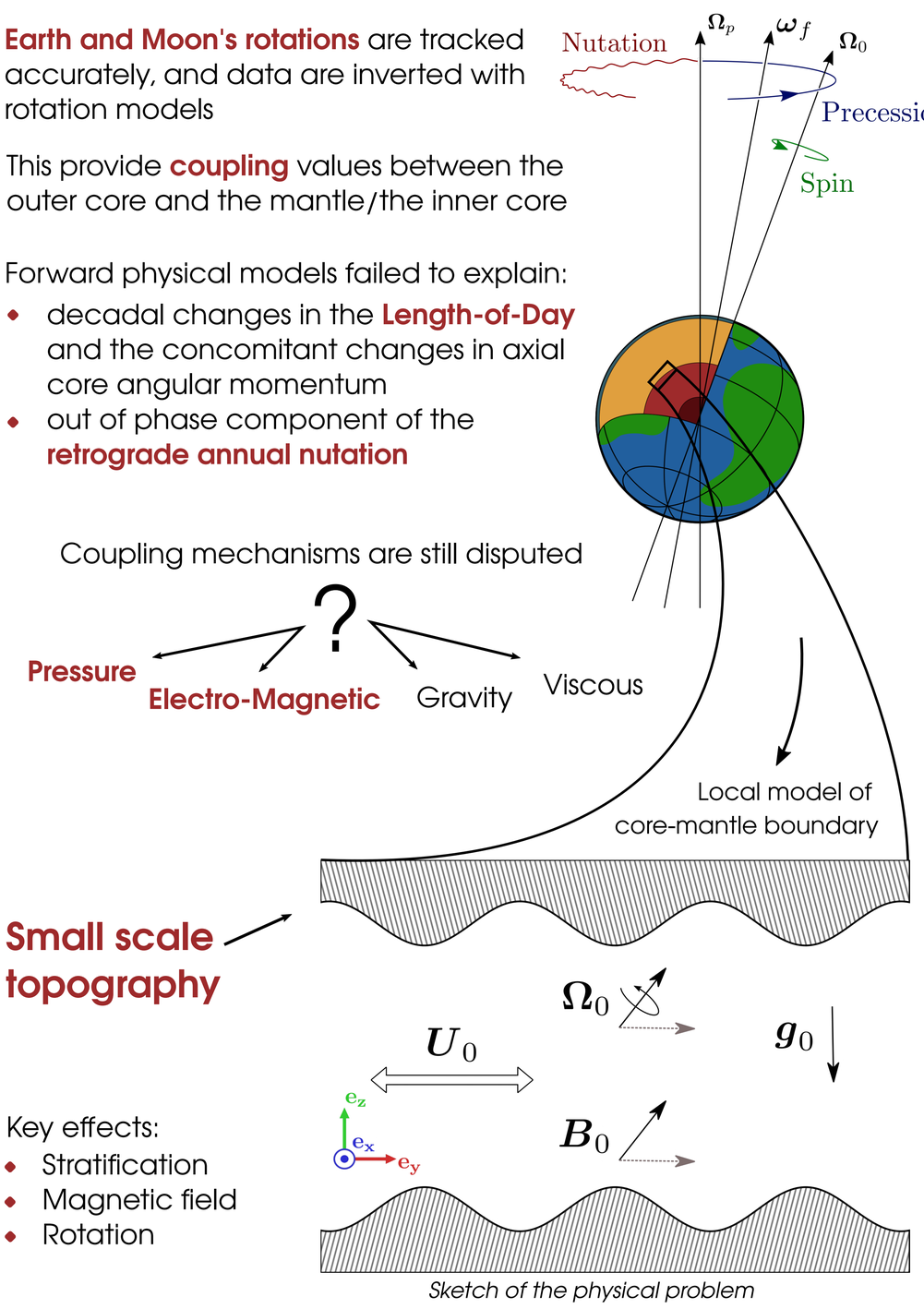
## Description of the problem

**Earth and Moon's rotations** are tracked accurately, and data are inverted with rotation models

This provide **coupling** values between the outer core and the mantle/the inner core

Forward physical models failed to explain:

- decadal changes in the **Length-of-Day** and the concomitant changes in axial core angular momentum
- out of phase component of the **retrograde annual nutation**

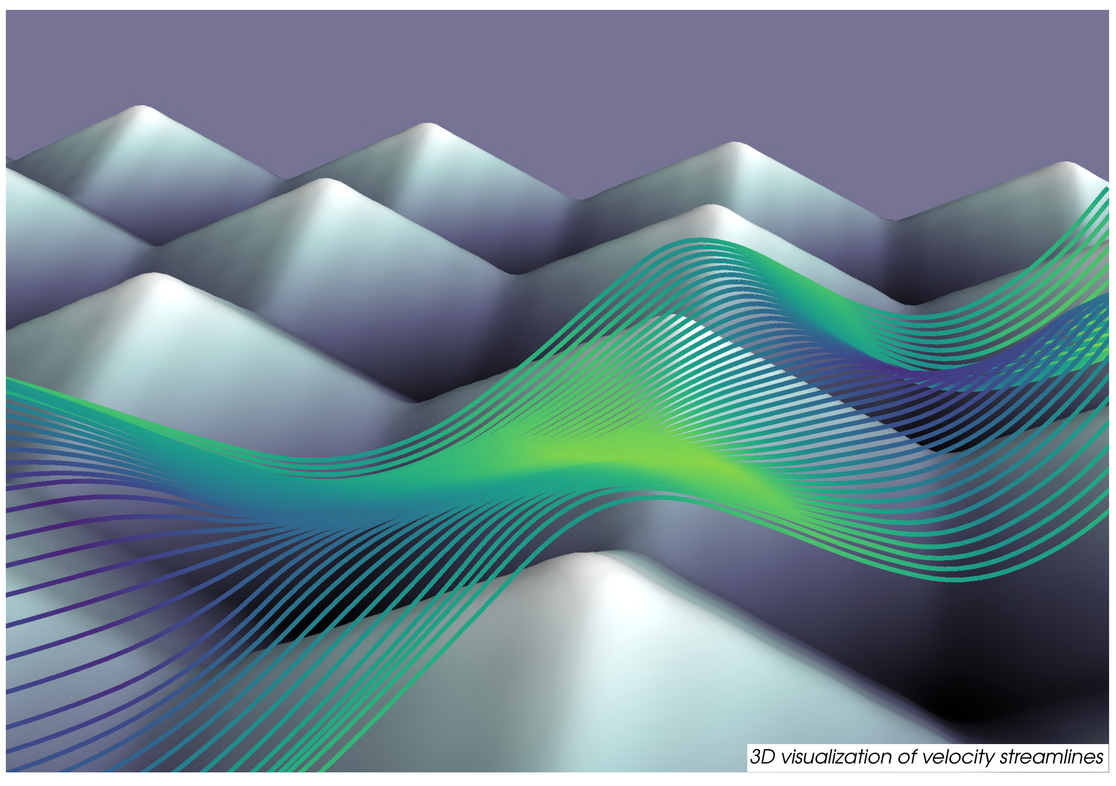



Key effects:

- Stratification
- Magnetic field
- Rotation

*Sketch of the physical problem*

Modelling the topographic **core-mantle coupling** is a long-standing issue dating back to Hide (1969)[1] or Braginsky (1998)[2].
By first following the recent work of Glane & Buffett (2018)[4] and Jault (2020)[5] on the LOD, we revisit the **Cartesian perturbative models** of **incompressible Boussinesq** hydro-magnetic flow over a **bumpy topography**, and then apply it in the case of the nutation, following Buffett (2010)[3].



*3D visualization of velocity streamlines*

## Methods

Our code **ToCCo** rely on symbolic and arbitrary precision calculations

2 steps

Derive the equations — Solve systems

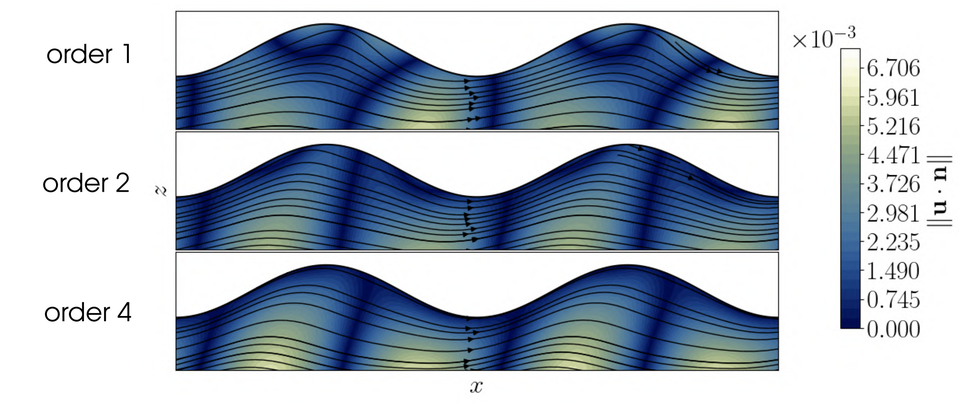
Sympy: Symbolic math
- Differentiation
- Taylor series
- Determinant of symbolic matrices

Arbitrary precision arithmetic
- Singular value decomposition
- Roots of polynomials

Equations are derived with a **perturbative** approach and **plane waves** approximation

We can explore **arbitrary topography** decomposed into Fourier series

Our "**higher-order**" solutions go beyond the **forced wave** linear regime by investigating **non-linear effects** which improves on previous results.




## Results

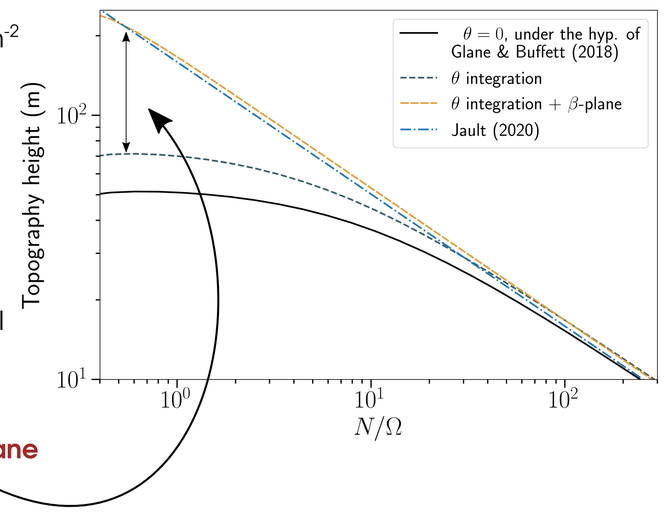
Data : application to length of the day

Nominal value for mean longitudinal stress[4] : 0.027 N.m$^{-2}$

Large modifications of calculated stress when including **variation of magnetic field and rotation vectors with the colatitude**

at order 0: **integration** of local values on the sphere

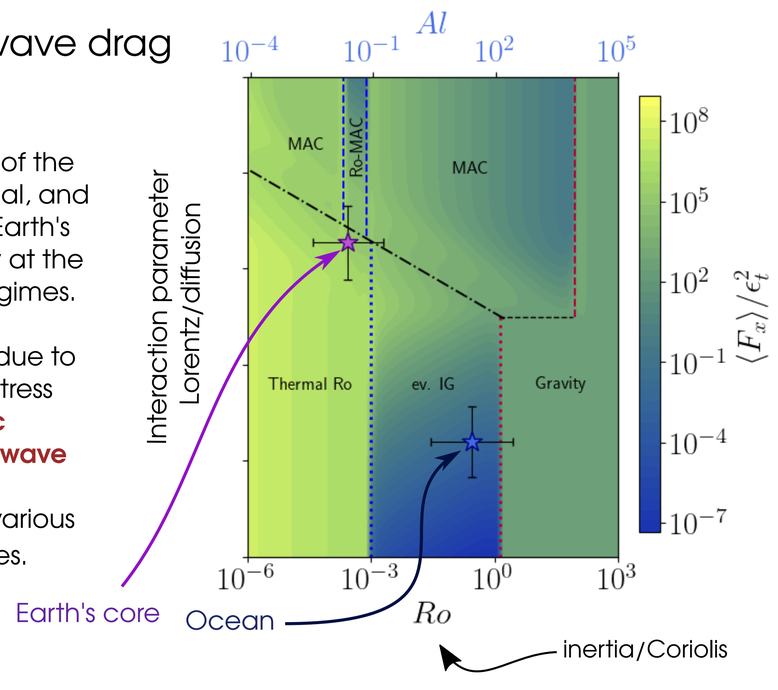
and order 1: **extended $\beta$ -plane effects** of Dellar (2011)[6]




### Topographic wave drag

Stress variations with physical parameters of the problem are non-trivial, and it turns out, that the Earth's core is unexpectedly at the frontiers of various regimes.

These variations are due to transitions between stress dominated by **ohmic dissipation** or by the **wave drag** due to the propagation of the various hydromagnetic waves.

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ToCCo:

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