

Topographic effects in magnetized and stratified fluid cores

Rémy Monville, David Cébron, Dominique Jault
Université Grenoble Alpes, CNRS, ISTerre, Grenoble, France

Description of the problem

Earth and Moon's rotations are tracked accurately, and data are inverted with rotation models

This provide **coupling** values between the outer core and the mantle/the inner core

Forward physical models failed to explain:

- decadal changes in the **Length-of-Day** and the concomitant changes in axial core angular momentum
- out of phase component of the **retrograde annual nutation**

Coupling mechanisms are still disputed

Pressure, Electro-Magnetic, Gravity, Viscous

Local model of core-mantle boundary

Stress-free mantle

Fluid core

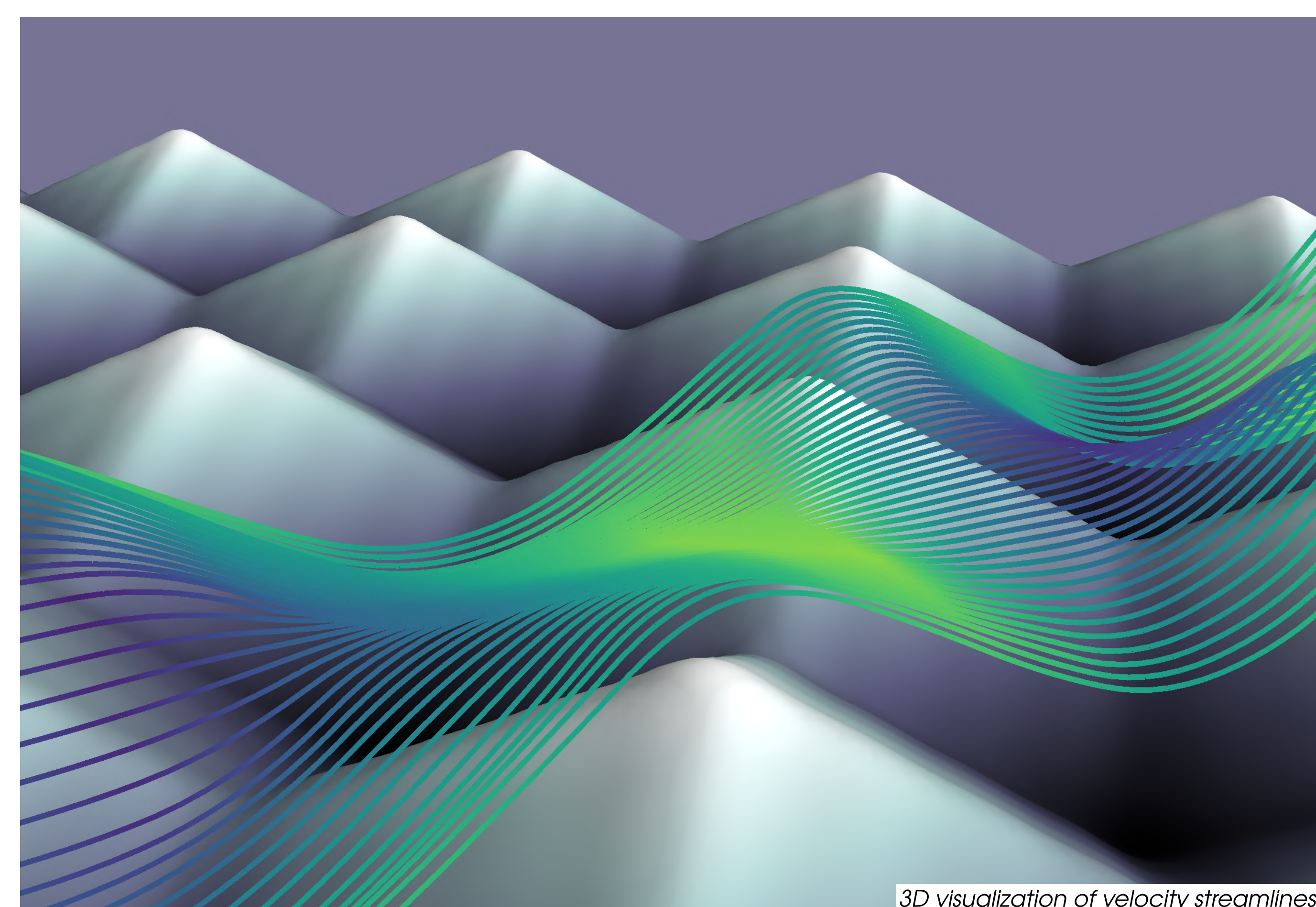
Small scale topography

Key effects:

- Stratification $\rho(z)$
- Magnetic field \mathbf{B}_0
- Rotation Ω

Sketch of the physical problem

We revisit the **Cartesian perturbative models** of incompressible **Boussinesq** hydro-magnetic flow over a **bumpy topography** following the works of Buffett (2010) on nutation, as well as Glane & Buffett (2018) and Jault (2020) on length of the day.



Methods

Our code **ToCCo** rely on symbolic and arbitrary precision calculations

Derive the equations → 2 steps → Solve systems

Sympy: Symbolic math

- Differentiation
- Taylor series
- Determinant of symbolic matrices

mpmath: Arbitrary precision arithmetic

- Singular value decomposition
- Roots of polynomials

Equations are derived with a **perturbative** approach and **plane waves** approximation

This allows to explore **arbitrary topography** shapes, decomposed into Fourier series

Our **"higher-order"** solutions go beyond the **forced wave** linear regime by investigating **non-linear effects** which improves on previous results.

Results

How high must the topography be to explain the length of the day data?

Mean stress = 0.027 N.m^{-2}

Large modifications of calculated stress when including **variation of magnetic field and rotation vectors with the colatitude θ**

at order 0: integration of local values on the surface

and order 1: extended β -plane effects of Dellar (2011)

From ocean to Earth's core: magnetic field effect on the coupling

Interaction parameter: magnetic forces

Non trivial scaling laws between geophysical problems

Strong variations due to transition between **ohmic dissipation** and propagations of the different **hydromagnetic waves**

Rossby Number: inertia/Coriolis