Topographic effects in magnetized and stratified fluid cores

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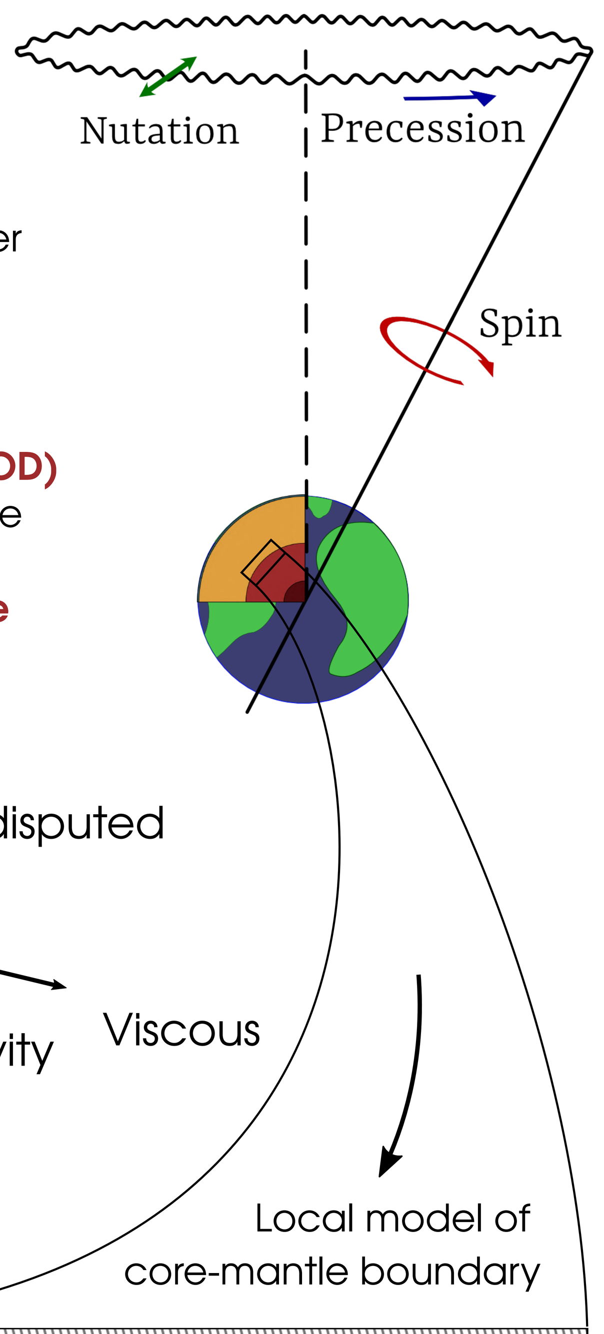
Description of the problem

Earth and Moon's rotations are tracked accurately, and data are inverted with rotation models

This provide **coupling** values between the outer core and the mantle/the inner core

Forward physical models failed to explain:

- decadal changes in the **Length-of-Day (LOD)** and the concomitant changes in axial core angular momentum
- out of phase component of the **retrograde annual nutation**



Coupling mechanisms are still disputed

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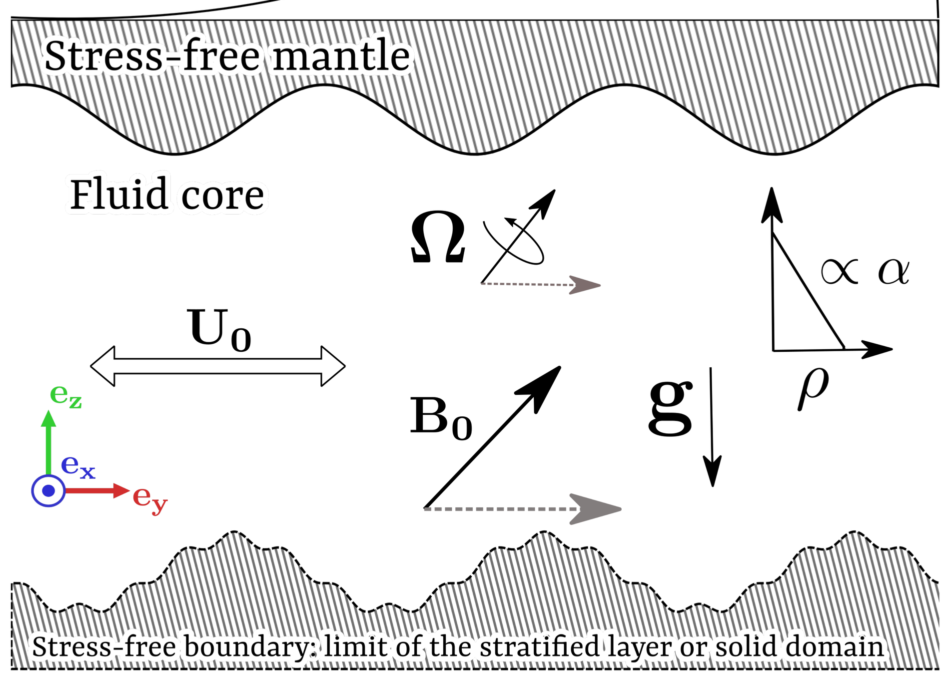
Pressure **Electro-Magnetic** Gravity Viscous

Local model of core-mantle boundary

Small scale topography



Sketch of the physical problem

Key effects:

- Stratification $\rho(z)$
- Magnetic field $\mathbf{B_0}$
- Rotation $\boldsymbol{\Omega}$

Modelling the topographic **core-mantle coupling** is a long-standing issue dating back to Hide (1969)[1] or Braginsky (1998)[2].
By first following the recent work of Glane & Buffett (2018)[4] and Jault (2020)[5] on the LOD, we revisit the **Cartesian perturbative models** of **incompressible Boussinesq** hydro-magnetic flow over a **bumpy topography**, and then apply it in the case of the nutation, following Buffett (2010)[3]. **Turbulent regimes** are planned to be explored experimentally in Grenoble and Zurich (J. Noir).

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Methods

Our code **ToCCo** relies on symbolic and arbitrary precision calculations

2 steps

Derive the equations — Solve systems

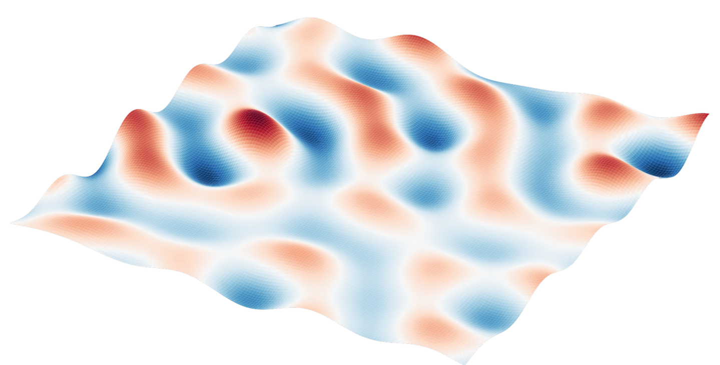
Sympy: Symbolic math
- Differentiation
- Taylor series
- Determinant of symbolic matrices

mpmath

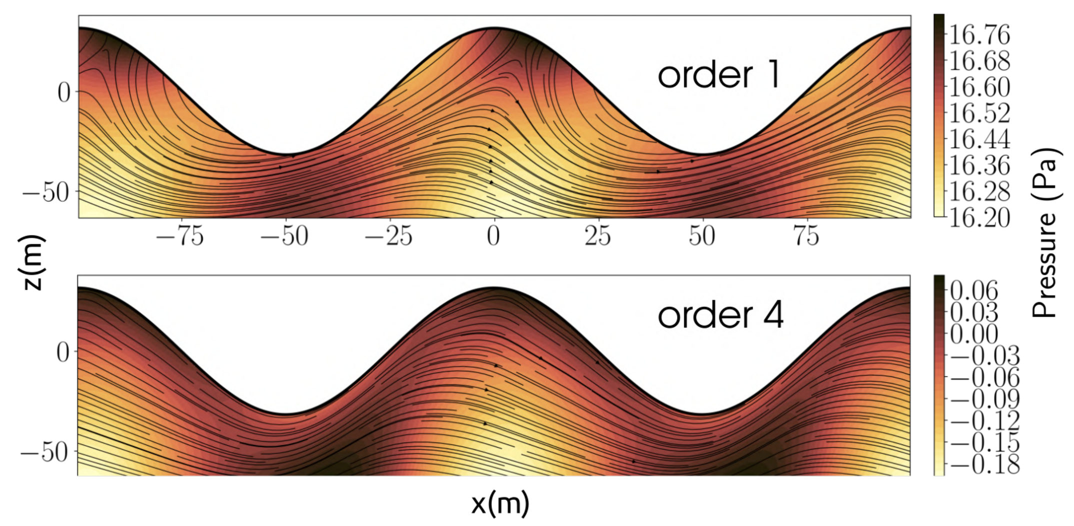
Arbitrary precision arithmetic
- Singular value decomposition
- Roots of polynomials

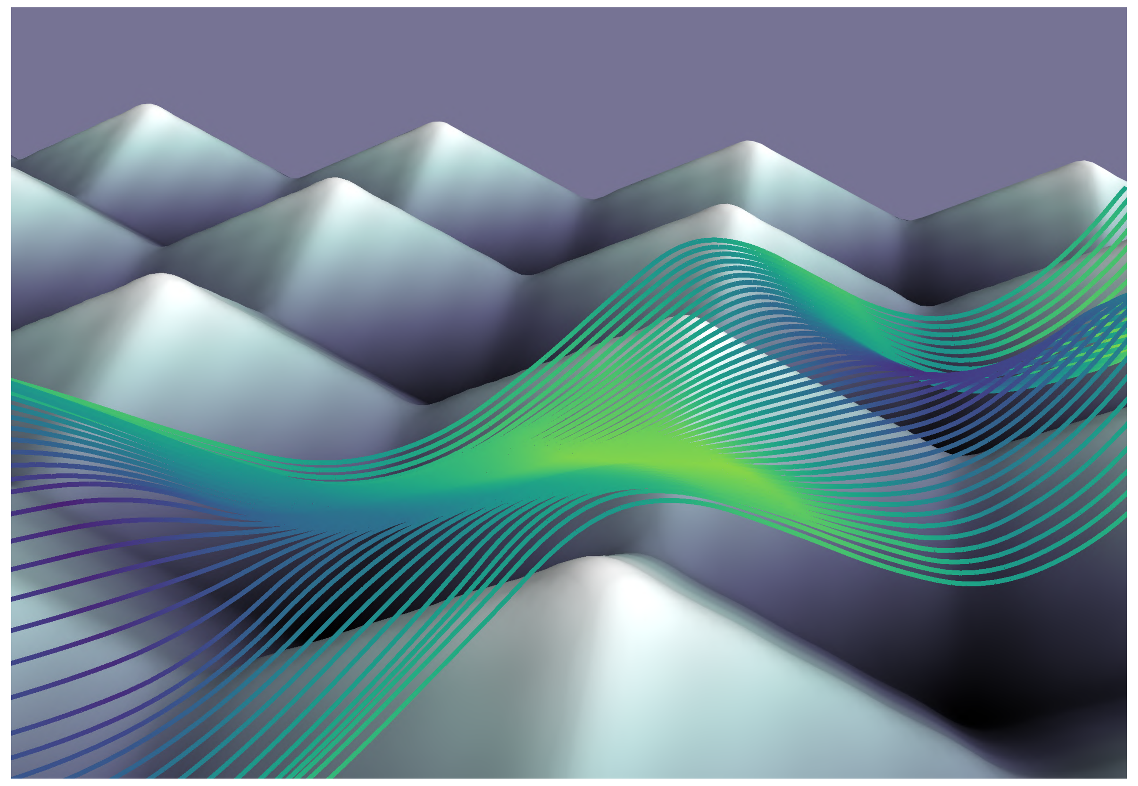
Equations are derived with a **perturbative** approach and **plane waves** approximation

We can explore **arbitrary topography** shapes, decomposed into Fourier series



Our **"higher-order"** solutions go beyond the **forced wave** linear regime by investigating **non-linear effects** which improve the previous results.



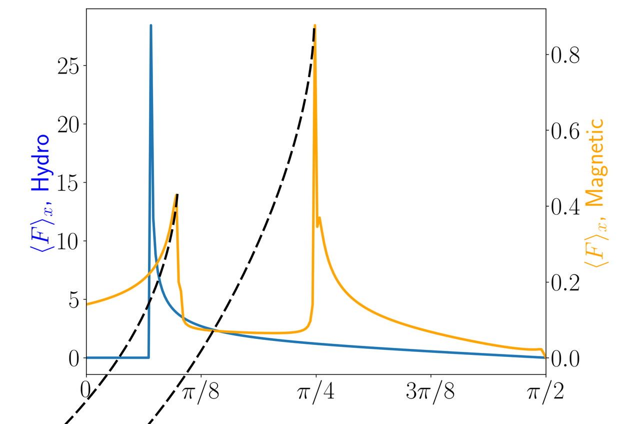


Results

Ohmic dissipation, wave drag and Rossby modes resonances

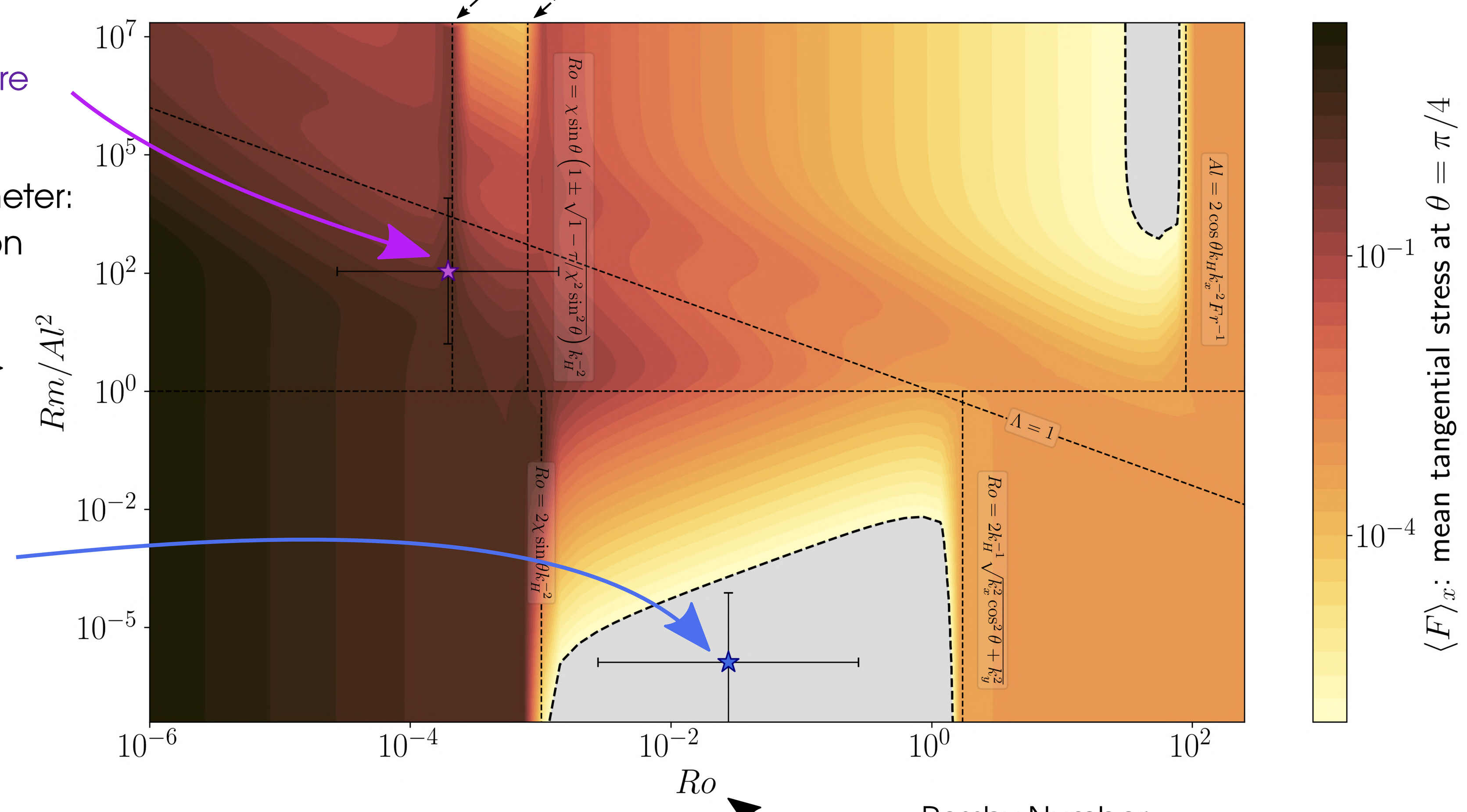
Stress variations with physical parameters of the problem are non-trivial, and it turns out, that the Earth's core is unexpectedly at the frontiers of various regimes.

These variations are due to transitions between stress dominated by **ohmic dissipation** or by the **wave drag** due to the propagation of the various hydromagnetic waves.



For small wavelength topography, the flow excites free Rossby modes.

At high *Rm*, the magnetic field **split** theses modes into two branches.



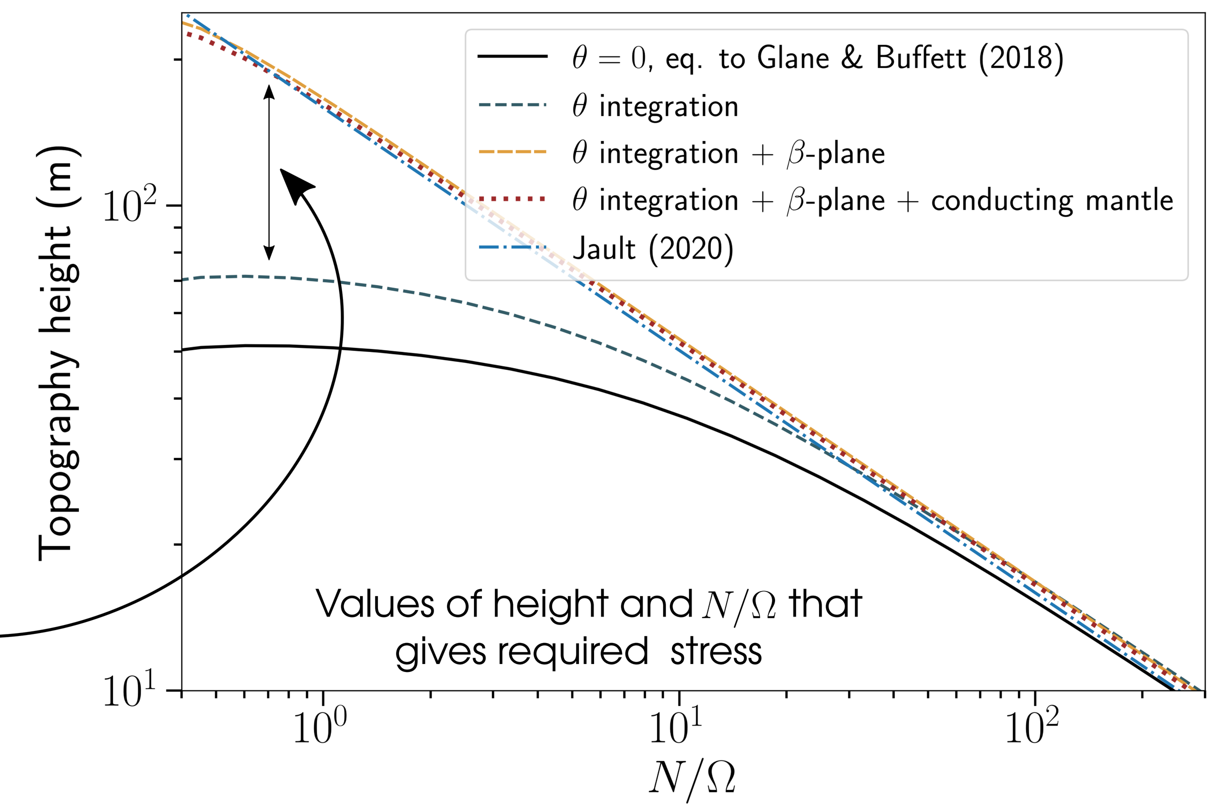
Data: application to LOD

Nominal value for mean longitudinal stress[4] : 0.027 N.m^{-2}

Large modifications of calculated stress when including **variation of magnetic field and rotation vectors with the colatitude**

at order 0: **integration** of local values on the sphere

and order 1: **extended β-plane effects** of Dellar (2011)[6]



[1] R. Hide, Nature 1969, 222, 1055.
[2] S. I. Braginsky, Earth, Planets and Space 1998, 50, 641.
[3] B. A. Buffett, Earth and Planetary Science Letters 2010, 296, 367.
[4] S. Glane, B. Buffett, Frontiers in Earth Science 2018, 6..
[5] D. Jault, Geophysical Journal International 2020, 221, 951.
[6] P. J. Dellar, Journal of Fluid Mechanics 2011, 674, 174.

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