Topography-driven flows in magnetized planetary layers
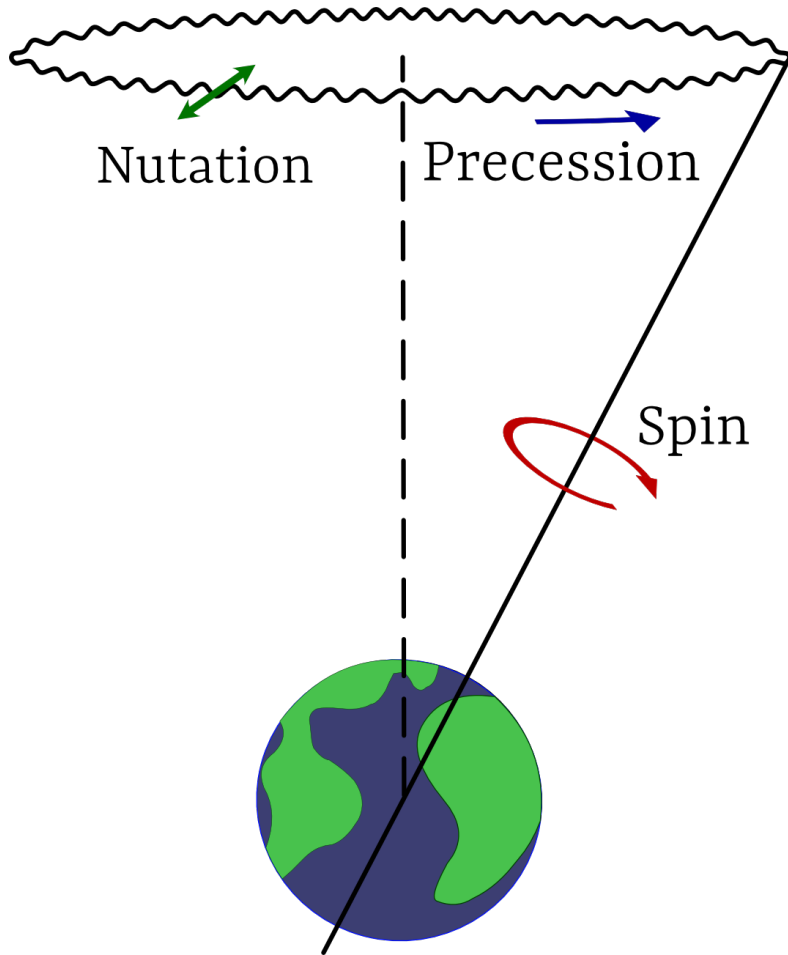
Rémy Monville, David Cébron, Dominique Jault
ISTerre, Université Grenoble Alpes, CNRS

22-23 Nov 2023
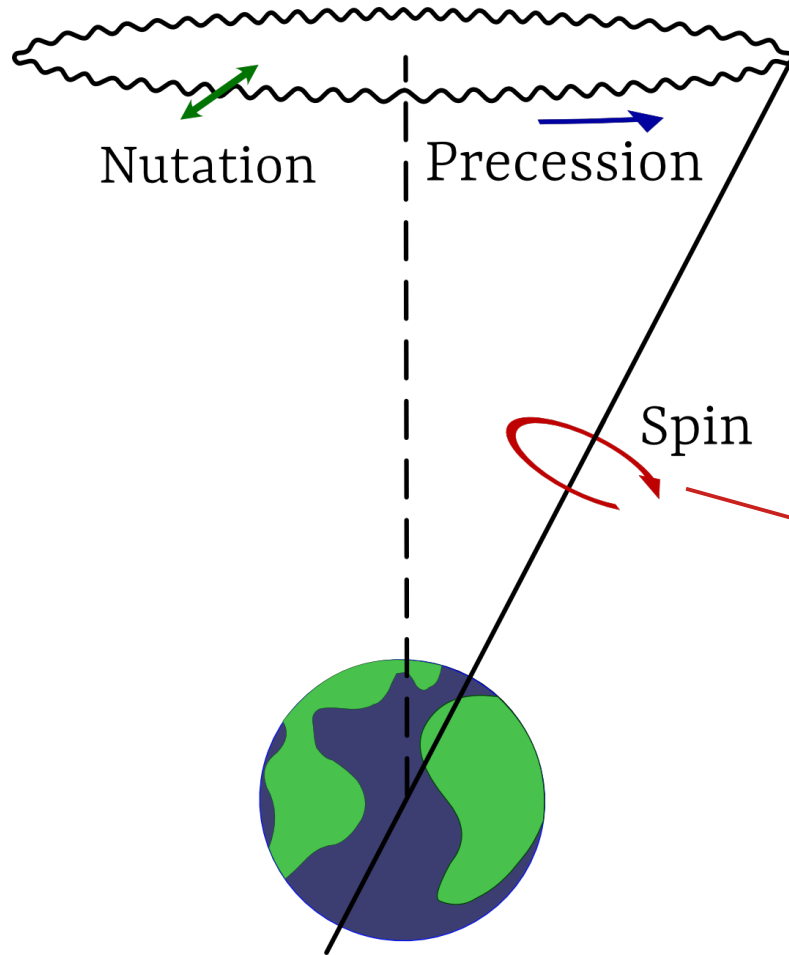**Graceful meeting**
Toulouse

Measure and model the Earth's rotation

Accurate measurements

The **forward models** failed to reproduce the data.

Measure and model the Earth's rotation
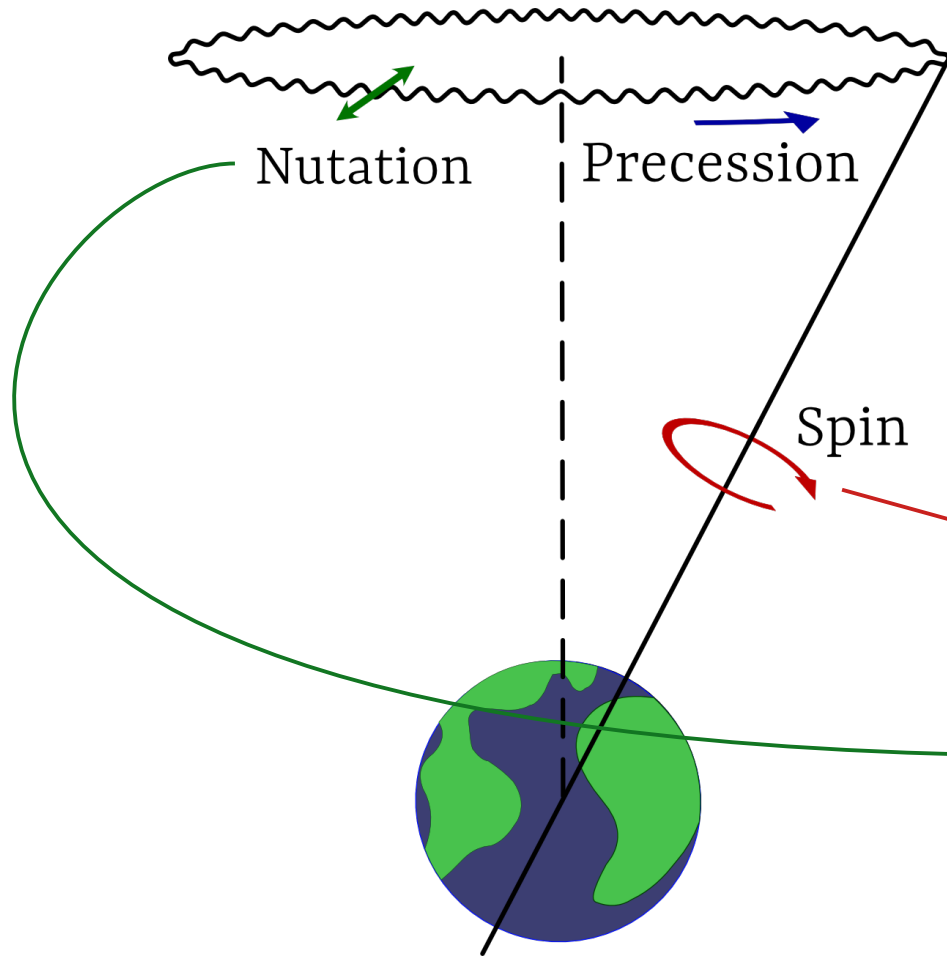

Nutation

Precession

Spin

Accurate measurements

The **forward models** failed to reproduce the data.

- Decadal changes in **Length of the day**

Measure and model the Earth's rotation

Nutation

Precession

Spin

Accurate measurements

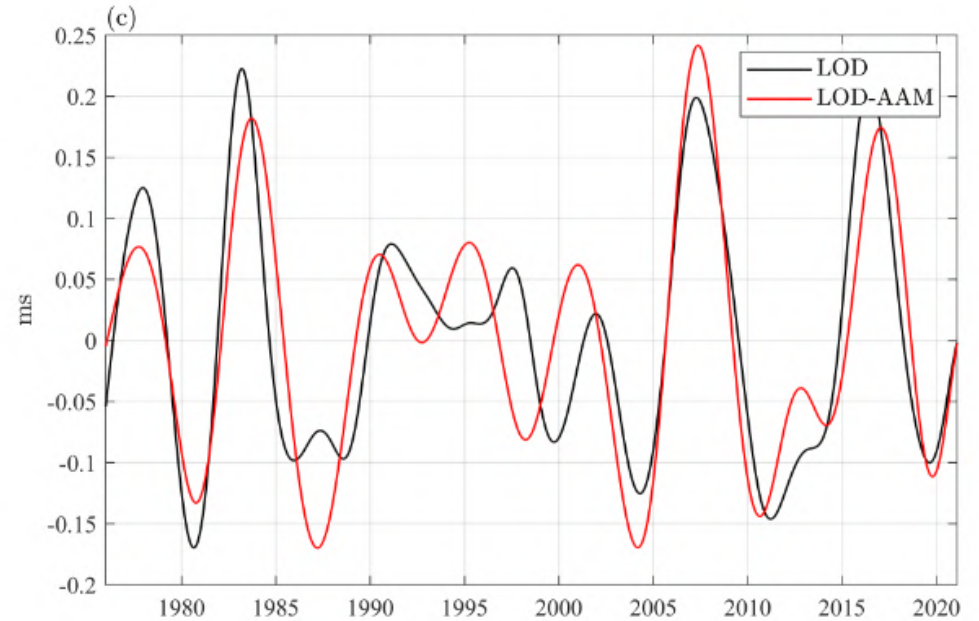The **forward models** failed to reproduce the data.

- Decadal changes in **Length of the day**

- Out of phase component in the retrograde **annual nutation**

Measure and model the Earth's rotation

Inversion of rotation data provide constraints on the **coupling between the liquid core and the mantle**.

Coupling mechanism are still disputed and struggles to explain all the nutation and LOD measurements **simultaneously**
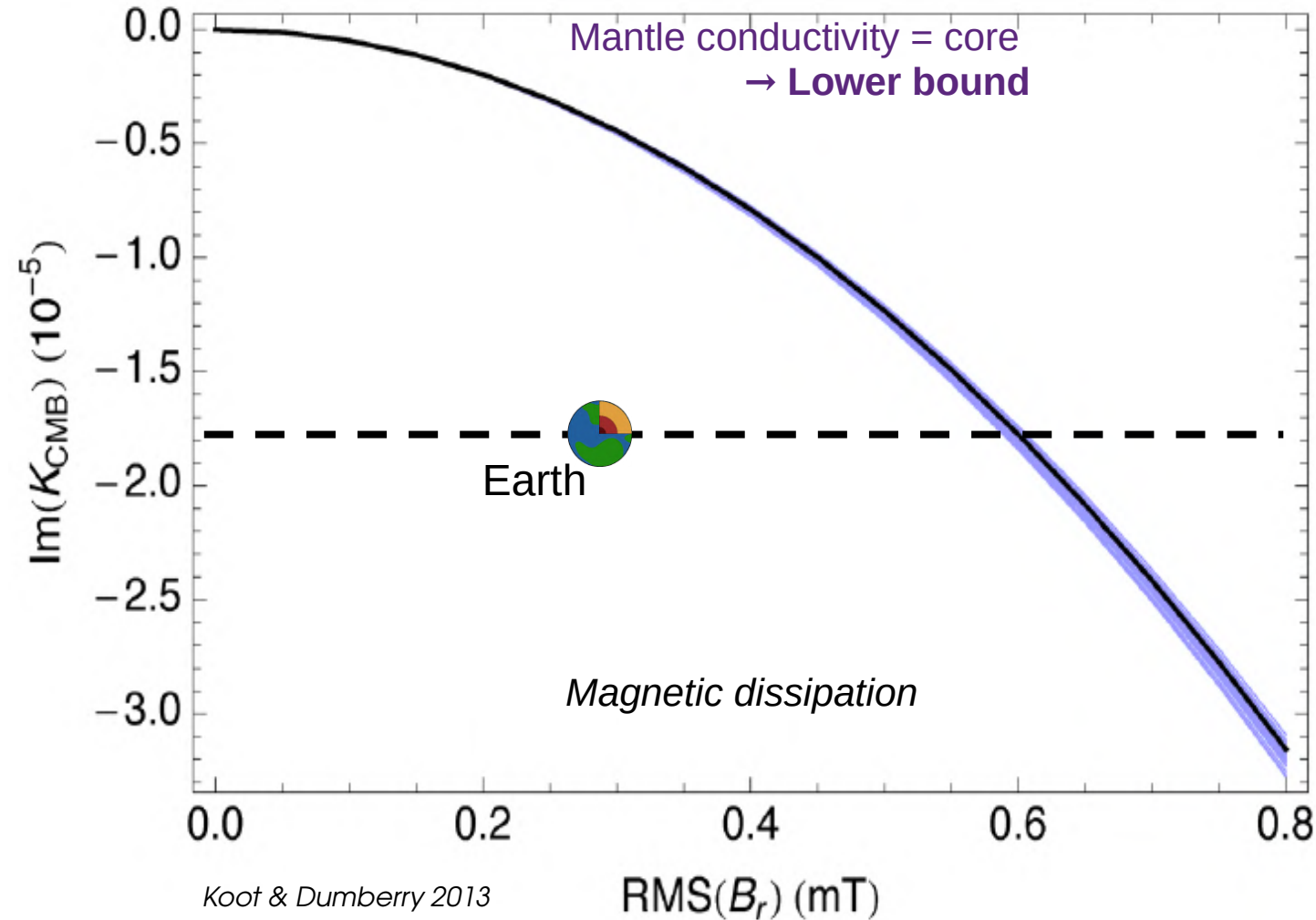

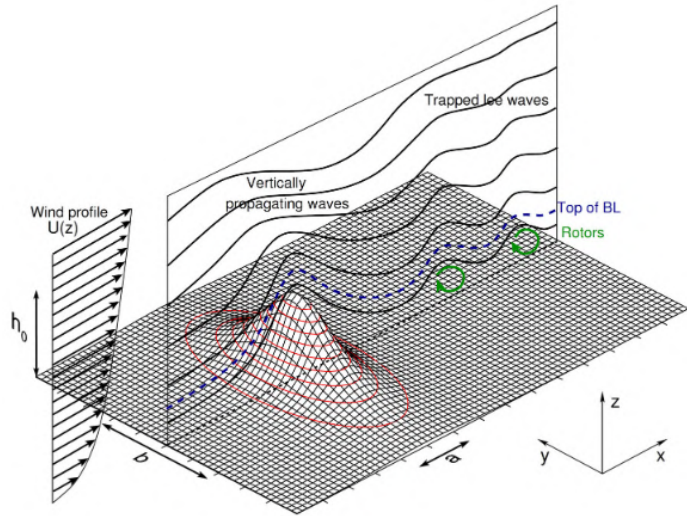*Rosat & Gillet 2023*

Fluid-solid couplings

Pressure   **Electromagnetic**   Viscous   Gravity

Earth core-mantle **dissipative** coupling

Mantle conductivity = core
→ **Lower bound**

Earth

Magnetic dissipation

Koot & Dumberry 2013

Electromagnetic coupling on flat boundary cannot explain the dissipation
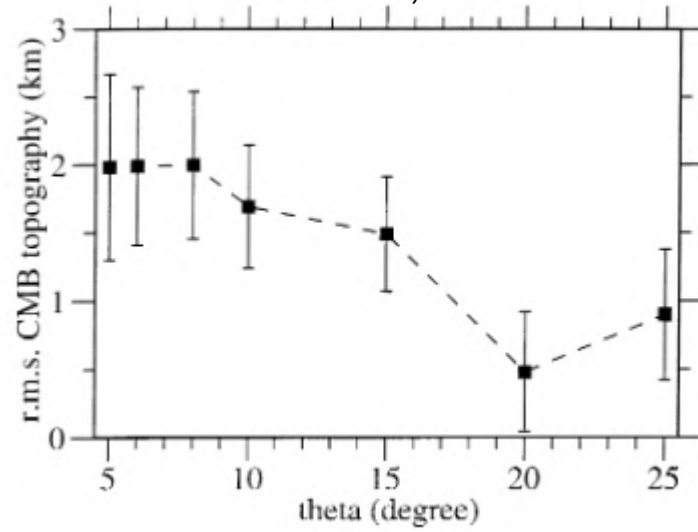
Need to add **topography** to increase this coupling

The topographic coupling, a generic features of geophysical flows

Garcia, R., & Souriau, A. (2000). Physics of the Earth and Planetary Interiors

Core

Subsurface oceans

Atmosphere

Oceans

M. A. Teixeira, Frontiers in Physics 2014, 2, 43.
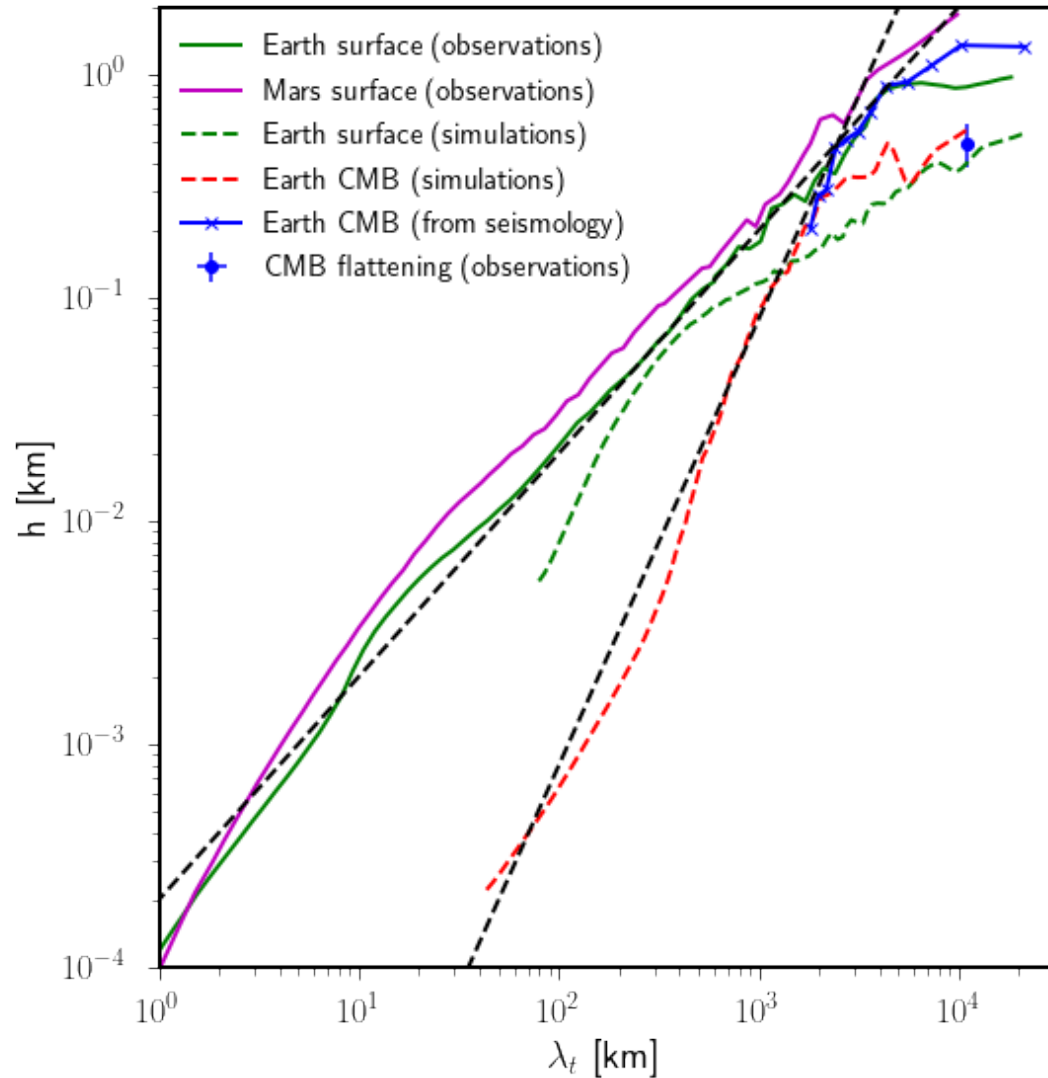
N. Balmforth, G. Ierley, W. Young, Journal of Physical Oceanography 2002, 32, 2900.
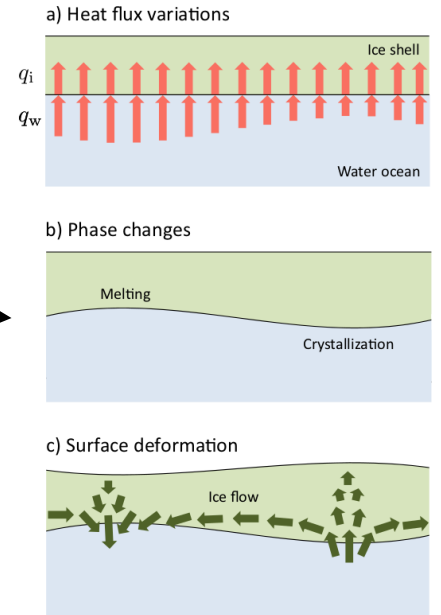
J. Kvorka, O. Čadek, G. Tobie, G. Choblet, Icarus 2018, 310, 149.
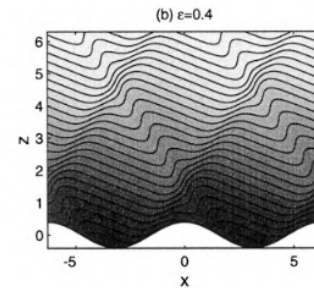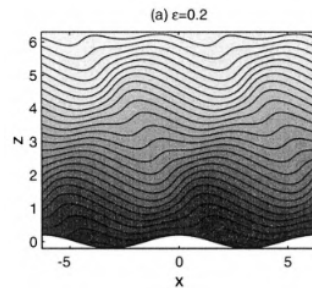
Is there a topography at the CMB ?

Legend:
- Earth surface (observations) — green solid
- Mars surface (observations) — magenta solid
- Earth surface (simulations) — green dashed
- Earth CMB (simulations) — red dashed
- Earth CMB (from seismology) — blue with x markers
- CMB flattening (observations) — blue point

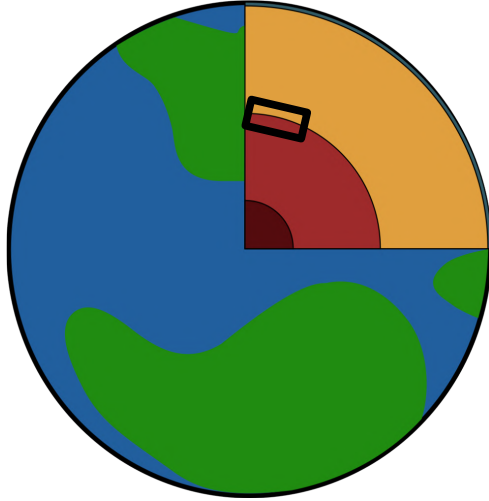Axes: h [km] vs λ_t [km]

CMB seismology: Koelemeijer 2021
Simulation: StagYY Simulations, courtesy of Thomas Frasson

Seismological studies suggest the existence of large scale topography

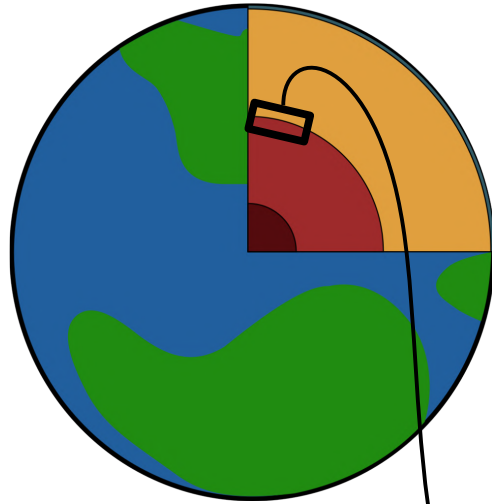**Mantle dynamics simulations** also give some insight on the smaller scales
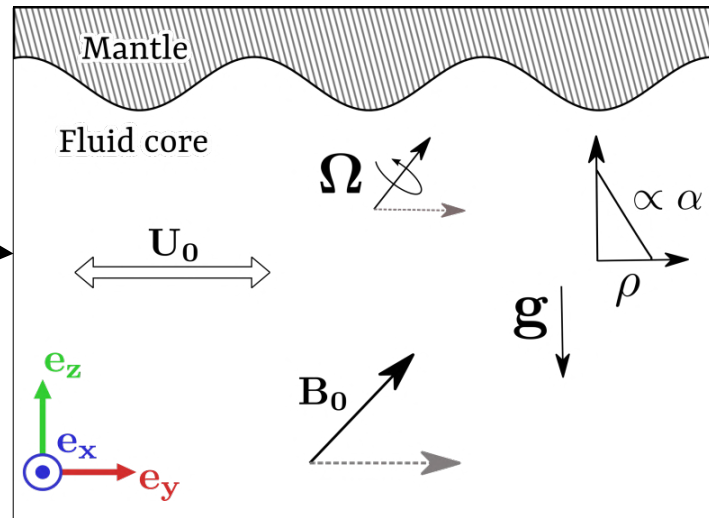
Both are in agreement

Method

How to model the topographic
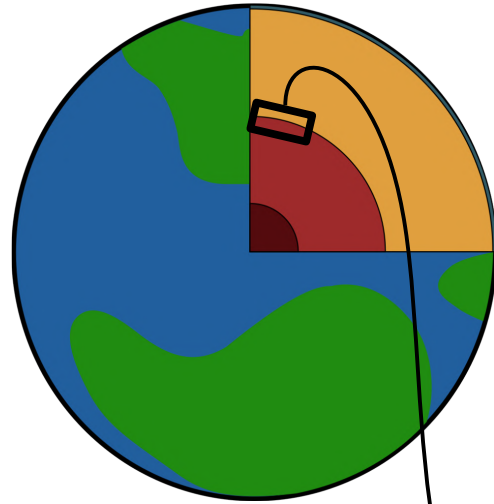coupling at the core mantle boundary ?
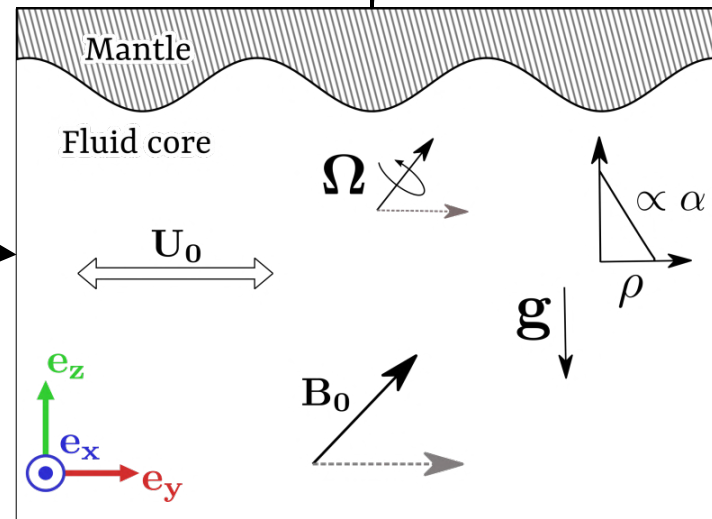
Method

Local model +
perturbation theory

Mantle

Fluid core

$\boldsymbol{\Omega}$

$\mathbf{U_0}$

$\propto \alpha$

ρ

$\mathbf{g}$

$\mathbf{B_0}$

$\mathbf{e_z}$

$\mathbf{e_x}$

$\mathbf{e_y}$

Key ingredients:
- Buoyancy
- Rotation
- Magnetic field

Method

Periodic topographies

Local model +
perturbation theory

Mantle

Fluid core

$\mathbf{\Omega}$

$\propto \alpha$

ρ

$\mathbf{U_0}$

$\mathbf{g}$

$\mathbf{B_0}$

$\mathbf{e_z}$

$\mathbf{e_x}$

$\mathbf{e_y}$
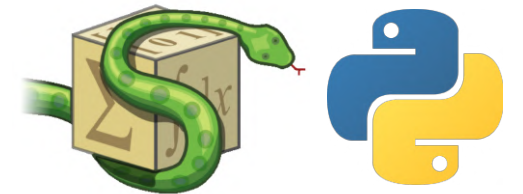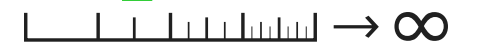
Key ingredients:
- Buoyancy
- Rotation
- Magnetic field
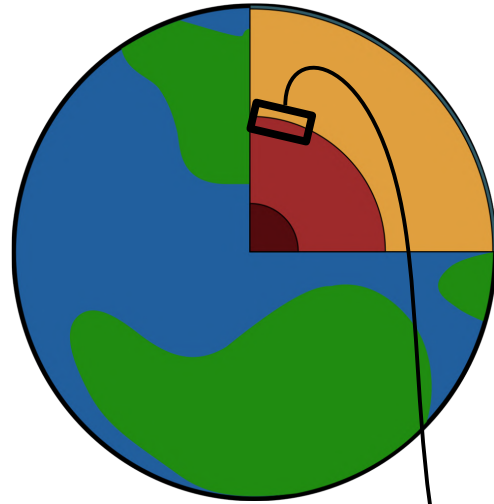
Method

Periodic topographies
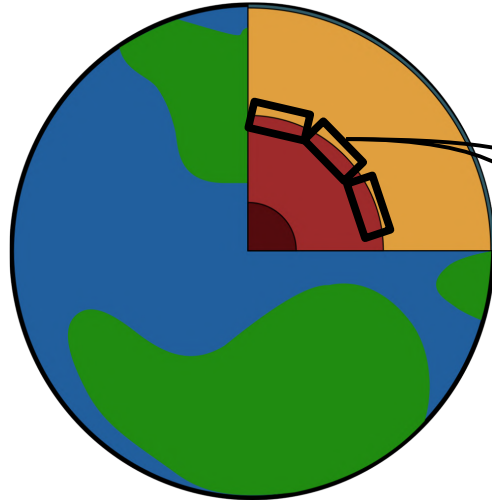

Local model +
perturbation theory
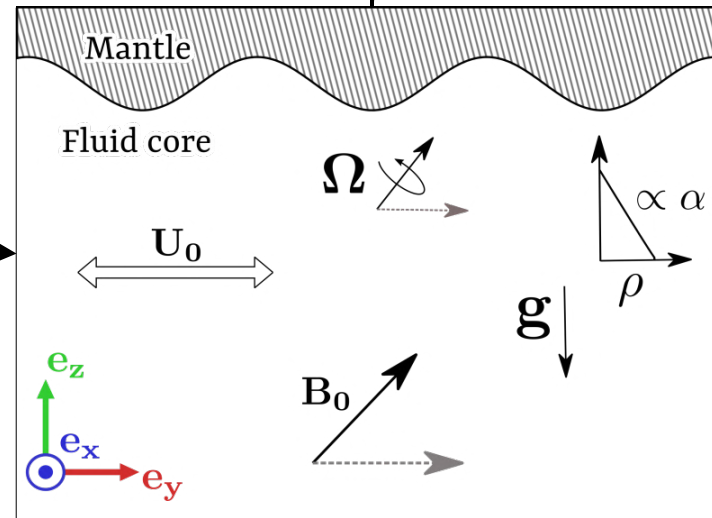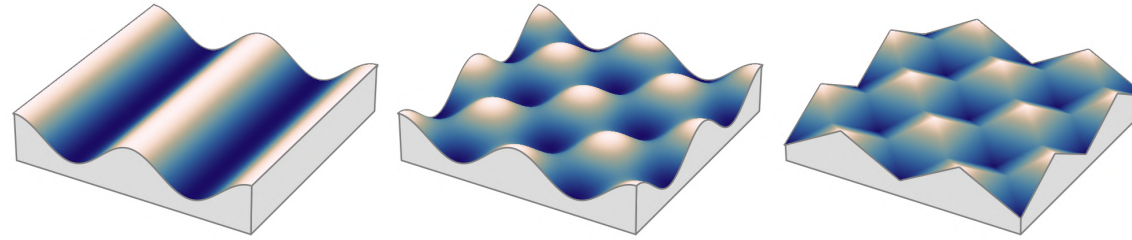
Symbolic calculation
+ arbitrary precision

Method

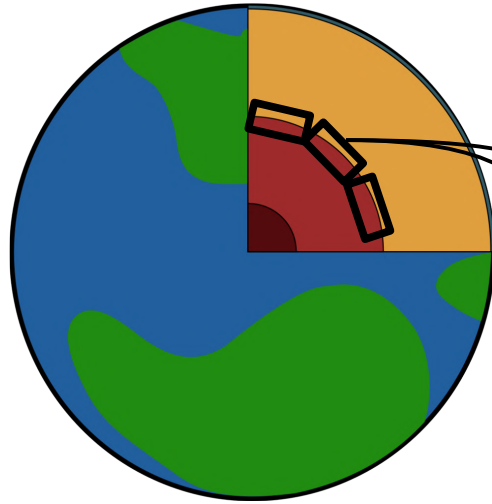## Effects of the global geometry

Multiple local boxes

0th order → uniform vectors

Method

Effects of the global geometry

Multiple local boxes

0th order → uniform vectors

dipole

$$\boldsymbol{B_0} = \left[0, \quad \boxed{\sin\theta/2}, \quad -\boxed{\cos\theta}\right],$$

$$\boldsymbol{\Omega} = \left[0, \quad \boxed{\sin\theta} \qquad , \quad \boxed{\cos\theta} \qquad\qquad\right],$$

Method

Effects of the global geometry
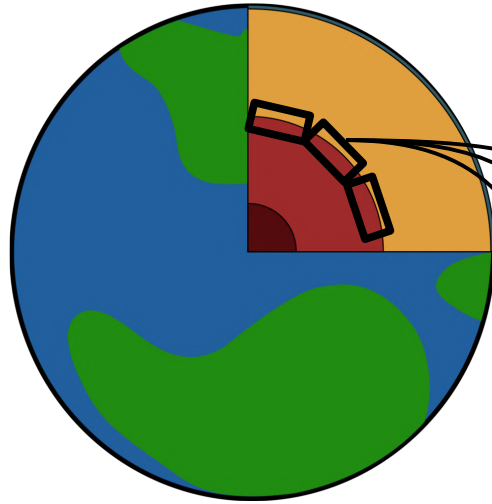
Multiple local boxes

0th order → uniform vectors

1st order → Beta-plane approximation

$$\boldsymbol{B_0} = \left[0, \quad \boxed{\sin\theta/2}, \quad -\boxed{\cos\theta}\right], \quad \rightarrow \text{ dipole}$$

$$\boldsymbol{\Omega} = \left[0, \quad \boxed{\sin\theta} - \boxed{2\chi y \cos\theta}, \quad \boxed{\cos\theta} + \boxed{2\chi z \cos\theta + \chi y \sin\theta}\right],$$

Method

Perturbation method at higher order

$$\boldsymbol{u} = \boldsymbol{u}_0 + \sum_{m,n=0} \epsilon_t^m \epsilon_v^n \boldsymbol{u}_{m,n}, \quad \boldsymbol{b} = Al^{-1}\boldsymbol{b}_0 + \sum_{m,n=0} \epsilon_t^m \epsilon_v^n \boldsymbol{b}_{m,n}, \quad a = \sum_{m,n=0} \epsilon_t^m \epsilon_v^n a_{m,n},$$

topography   non-linearities
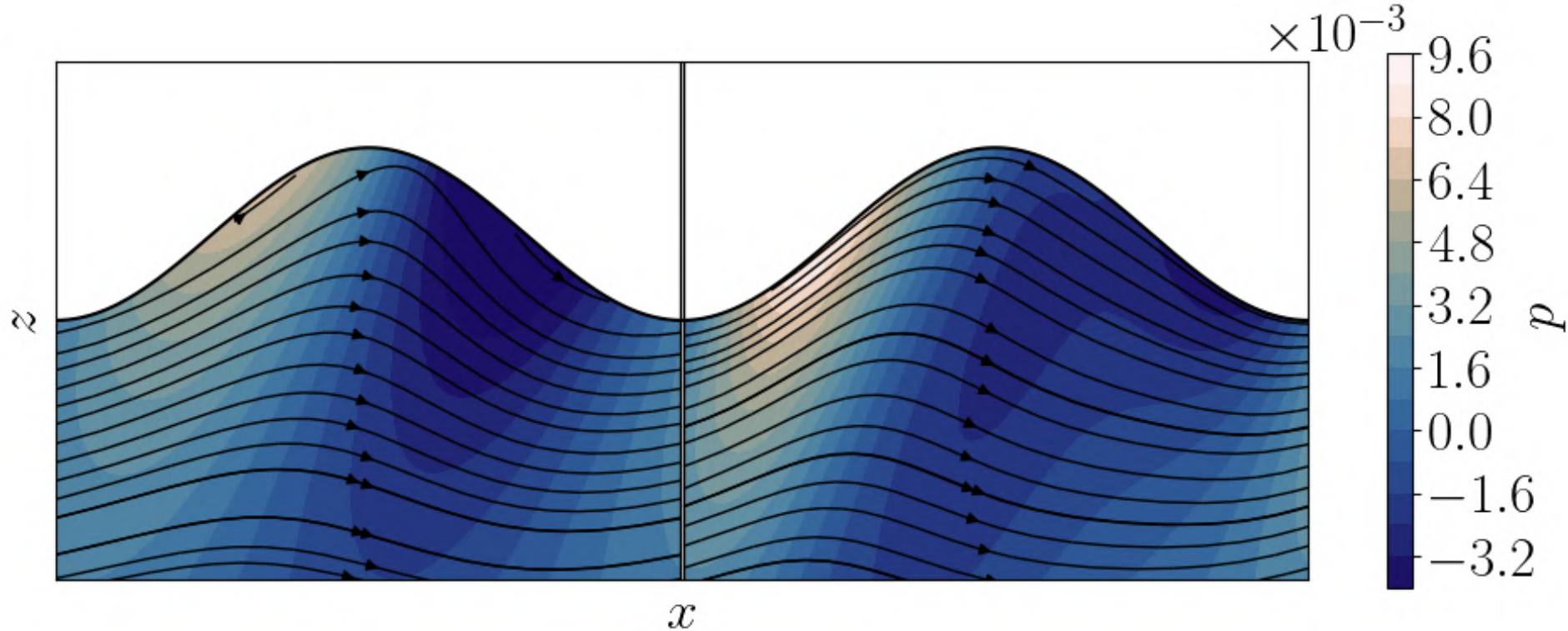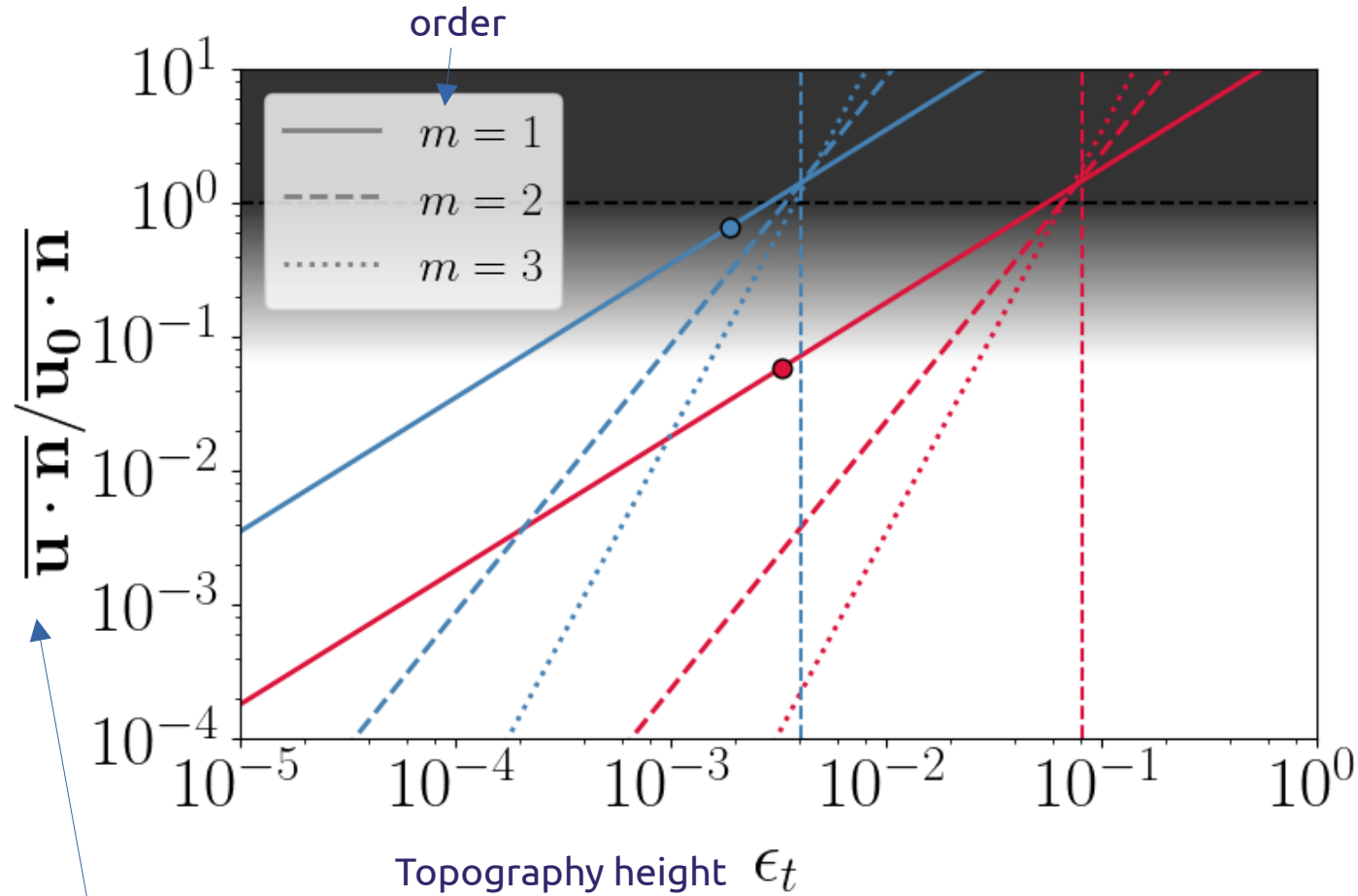

→ Increase accuracy   1st order   4th order

Method
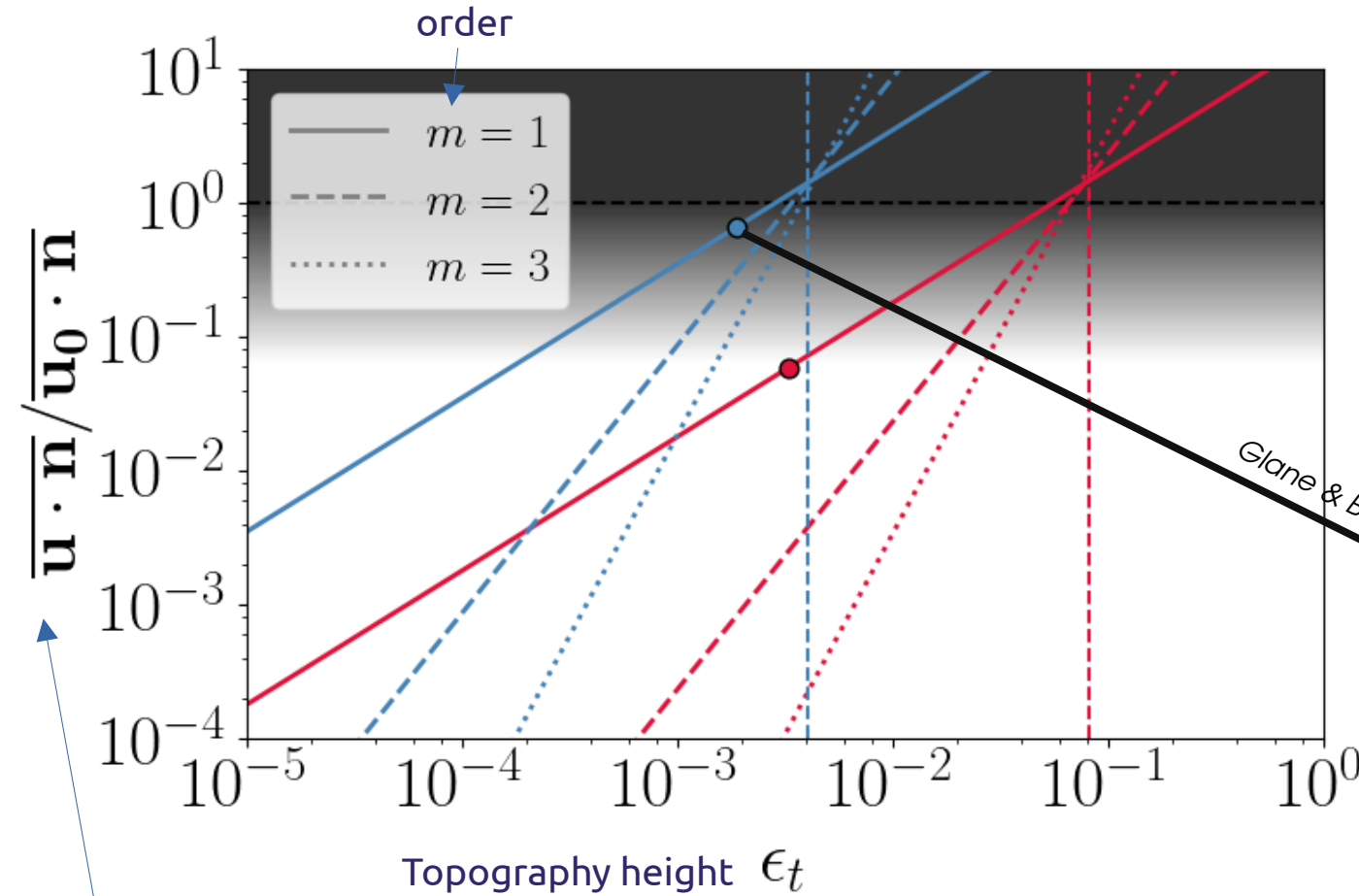
Perturbation method at high order

← Bounds the convergence of the model

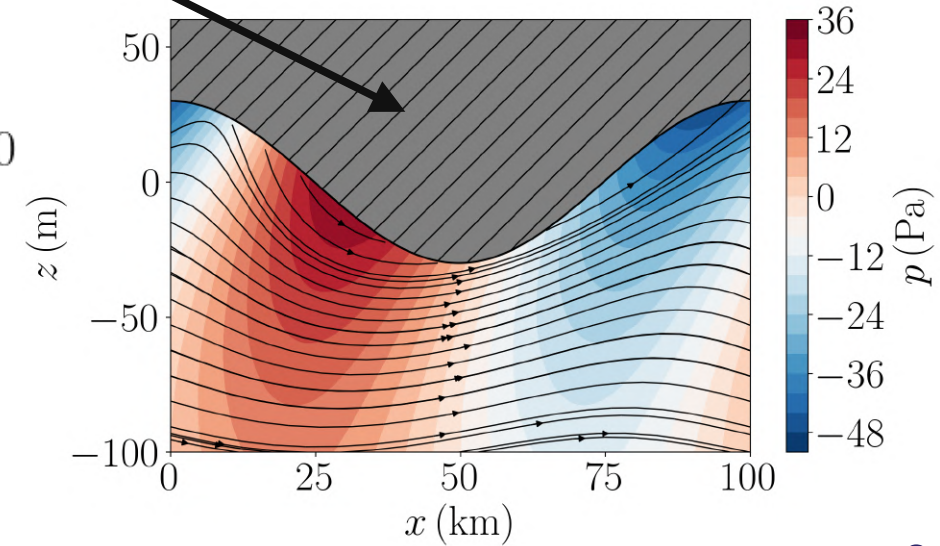Error on the non-penetration boundary condition

Method

Perturbation method at high order

order

← Bounds the convergence of the model

Glane & Buffett (2018)

$\overline{\mathbf{u}\cdot\mathbf{n}}/\overline{\mathbf{u}_0\cdot\mathbf{n}}$

$m = 1$
$m = 2$
$m = 3$

Topography height ϵ_t

Error on the non-penetration boundary condition

$z\,(\mathrm{m})$

$x\,(\mathrm{km})$

$p\,(\mathrm{Pa})$

Does pressure stress vary differently within the various planetary fluid layers

f-plane   β-plane

Monville, Cébron & Jault, Submiitted
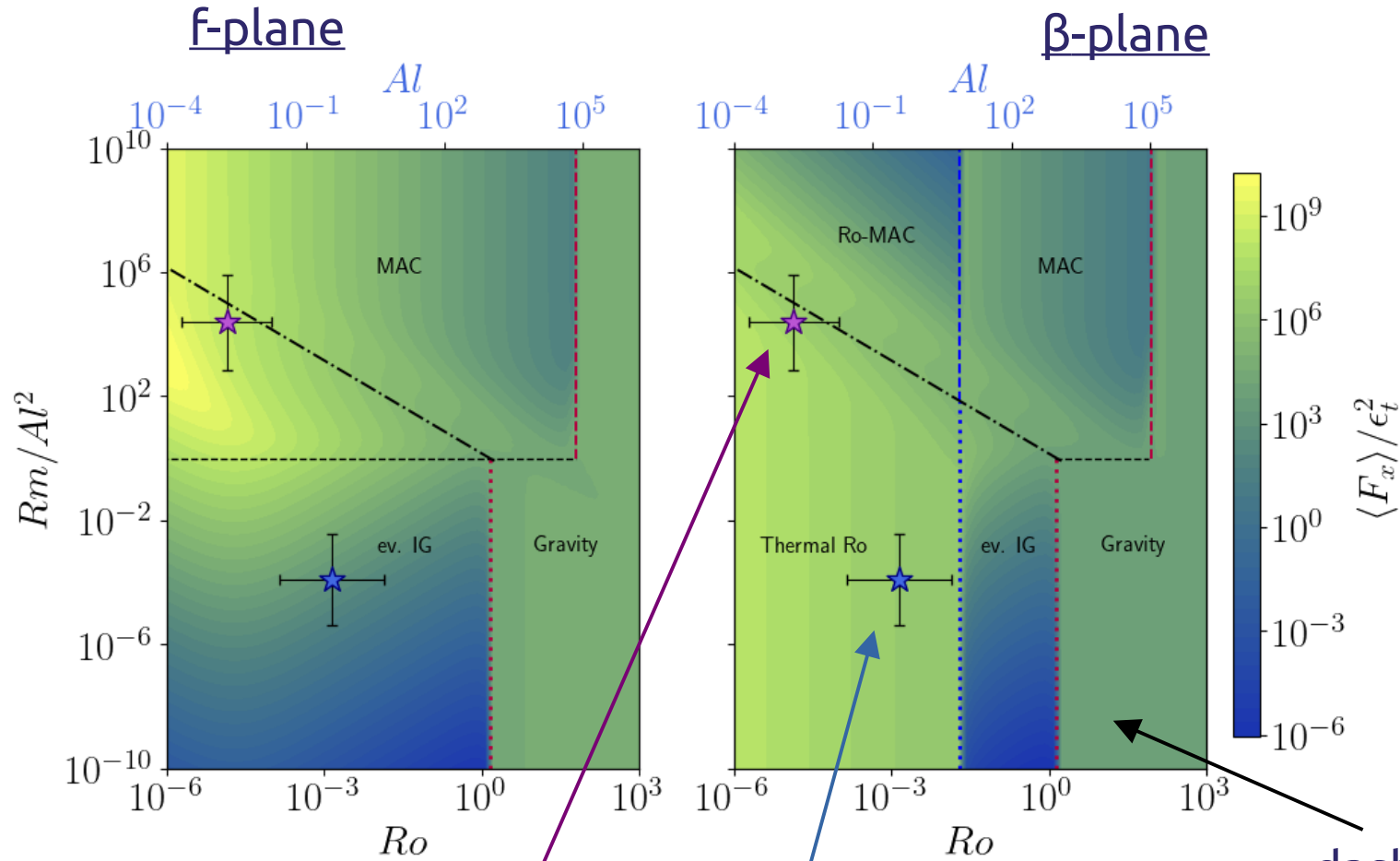
Earth's core   Ocean

Two key effects:

- **Rotation:** stress highly impacted by β effects

- **Magnetic field**
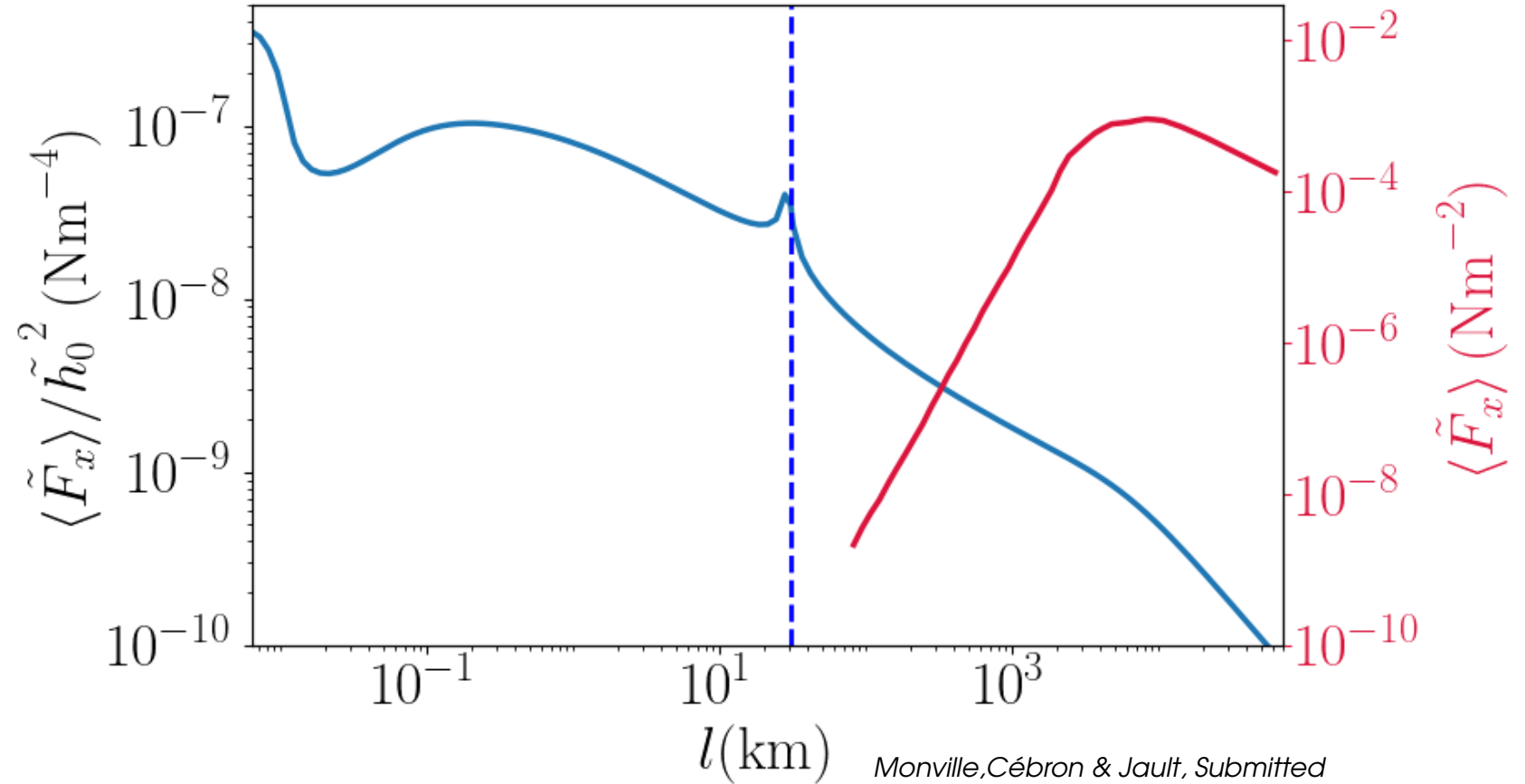
dashed/dotted lines = our analytical predictions (from MHD wave eq.) of the regimes

Back to data : Scale dependence of the stress

CMB topography model

Monville, Cébron & Jault, Submitted

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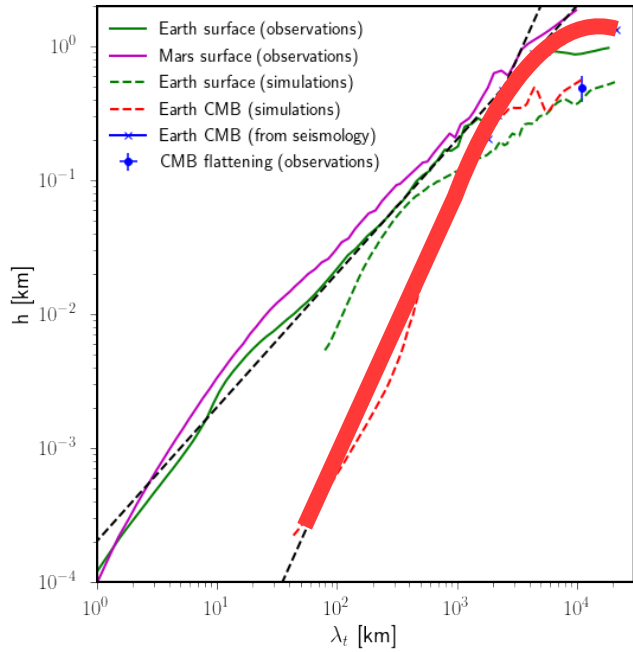
Back to data : Length of the day
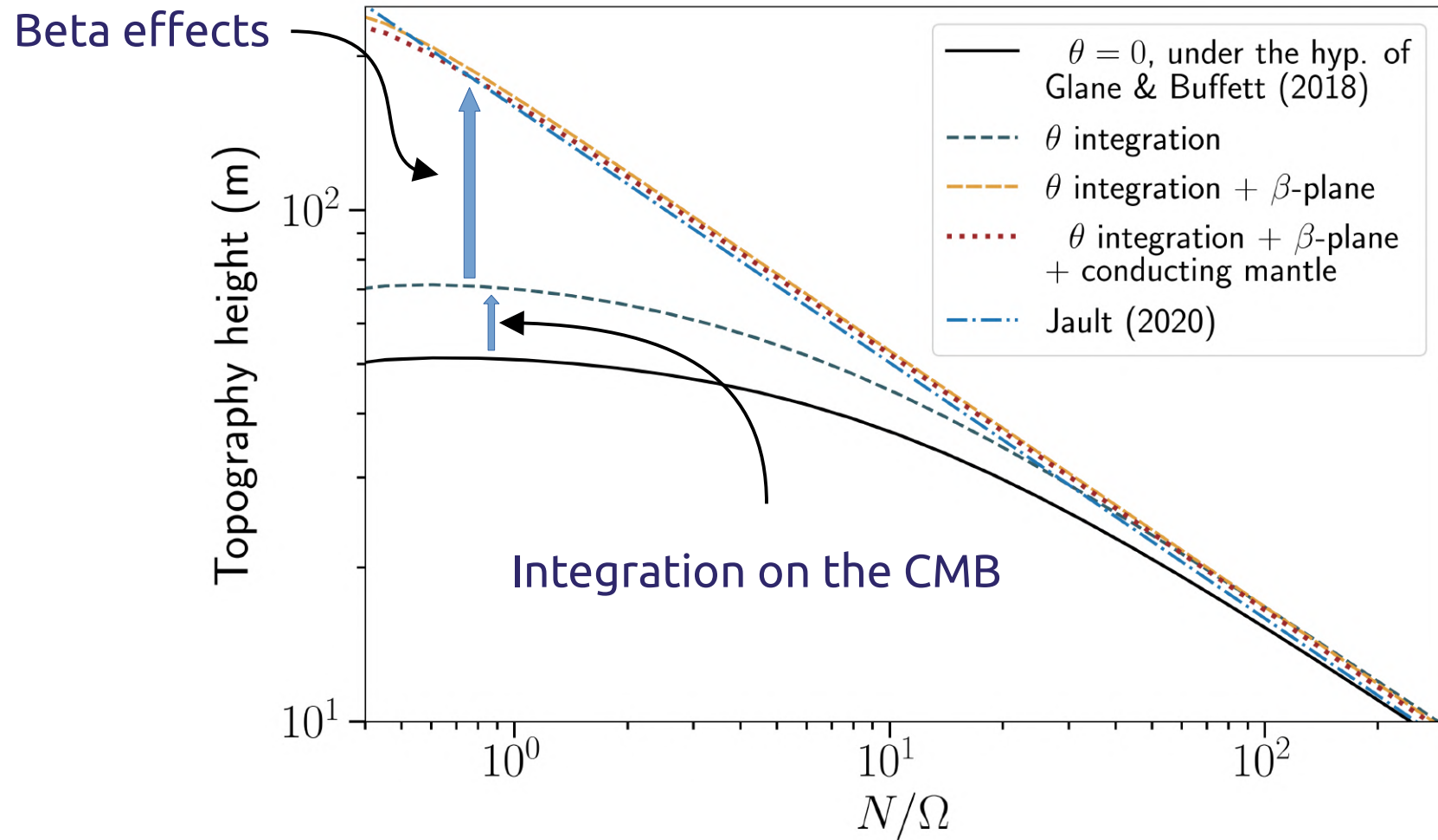
With these variations and **our new features** we expect to **reconcile length of day and nutation data**.

Conclusion

- We developed a **robust model** able to solve many types of problem **efficiently**.

- **High order** perturbation : better **accuracy** & provides the **limits** of this method

- New insight on the topography **coupling** and **topographic waves** in the Earth core context

- Automated method: easy to **optimise on geophysical data**.

- In the future, we plan to link up with **rotating table experiments**.

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