Rotating convection in stably-stratified planetary cores

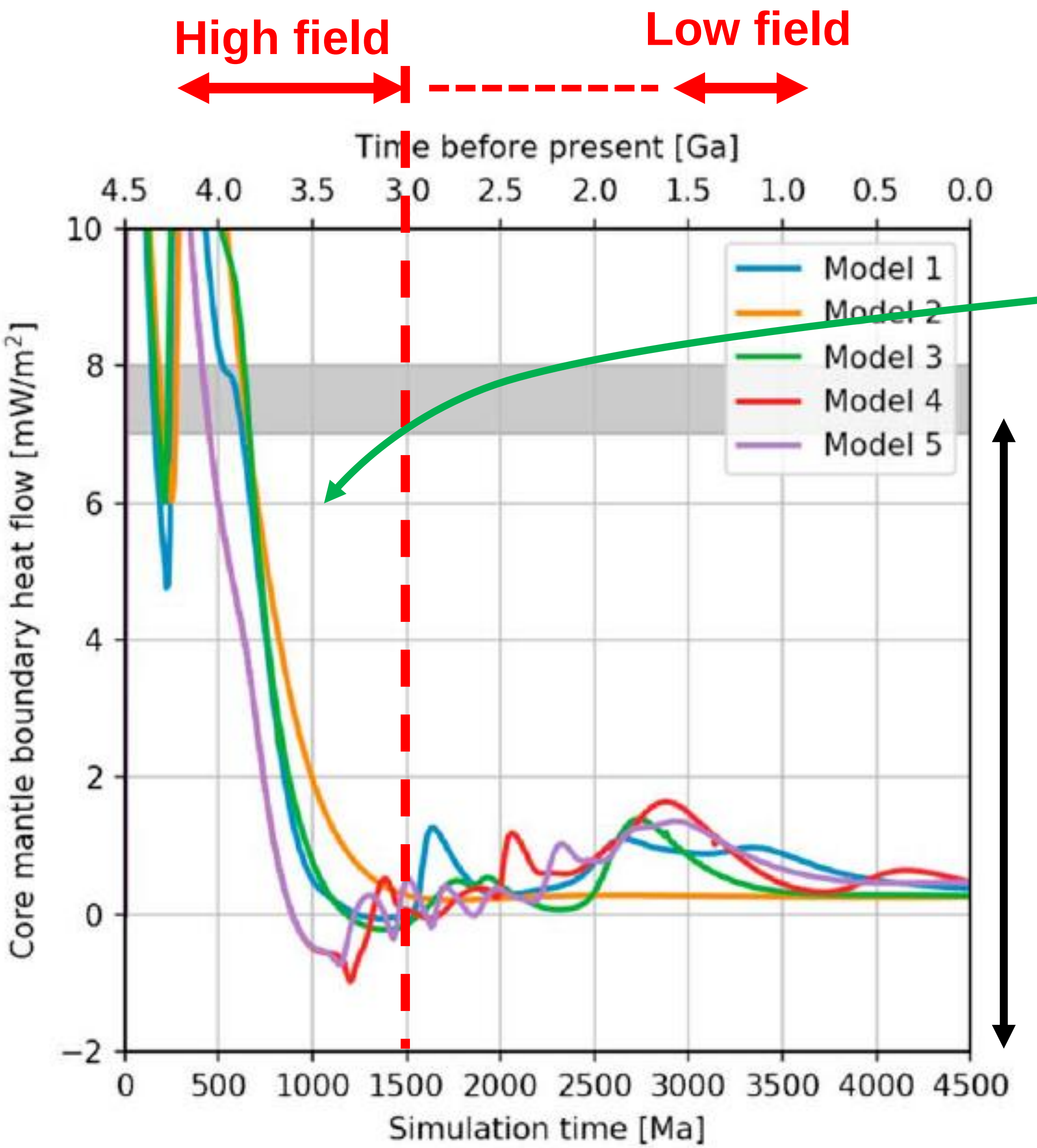
R. Monville, J. Vidal, D. Cébron, N. Schaeffer

ISTerre, CNRS, Univ. Grenoble Alpes

The Core of the Moon, Marseille, May 20-22, 2019

A thermally stratified lunar core?

Laneuville et al. JGR 2018

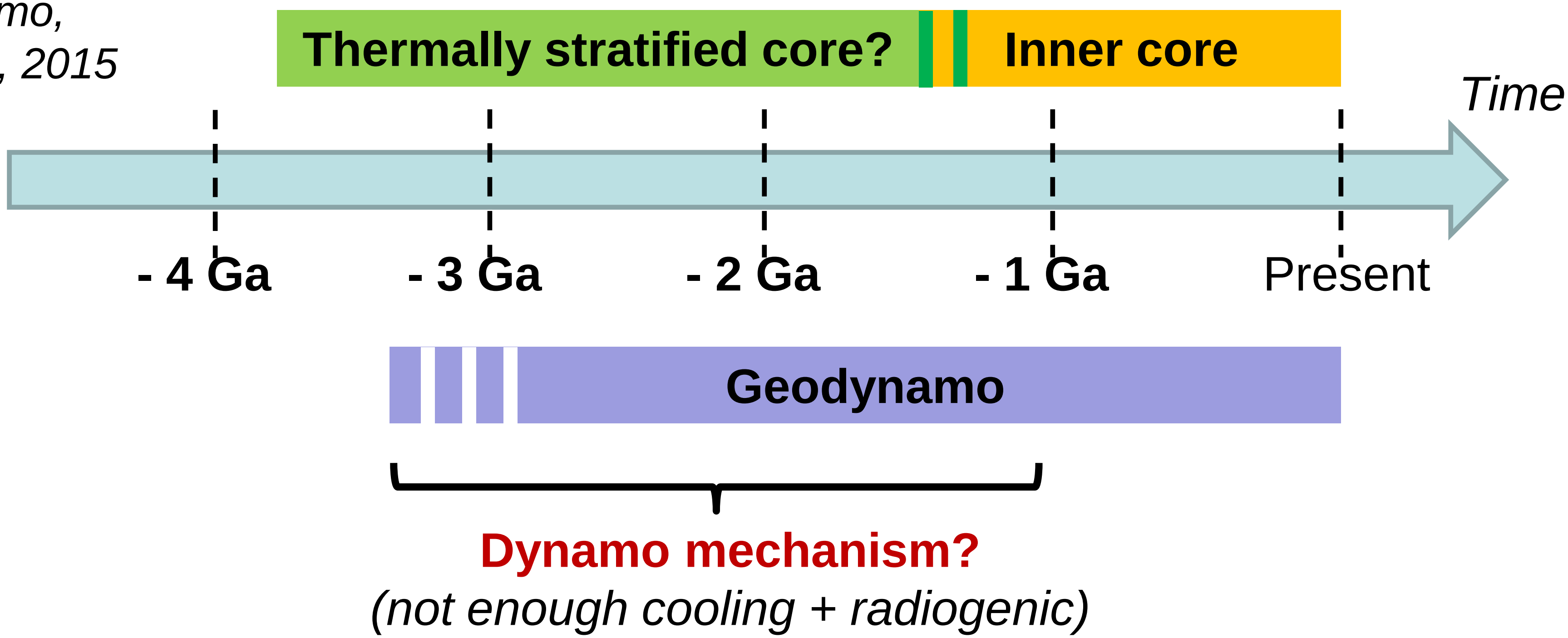


Lunar dynamo in a stably-stratified core?

Sub-adiabatic lunar core = stratified lunar core

A thermally stratified Earth's core?

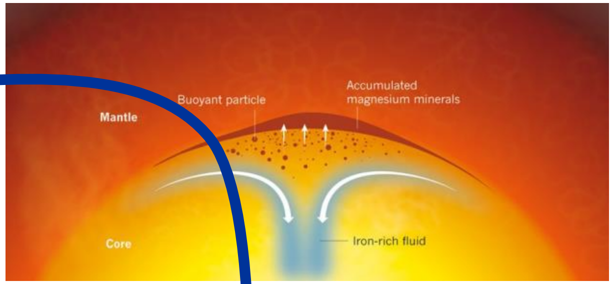
Nimmo, ToG, 2015



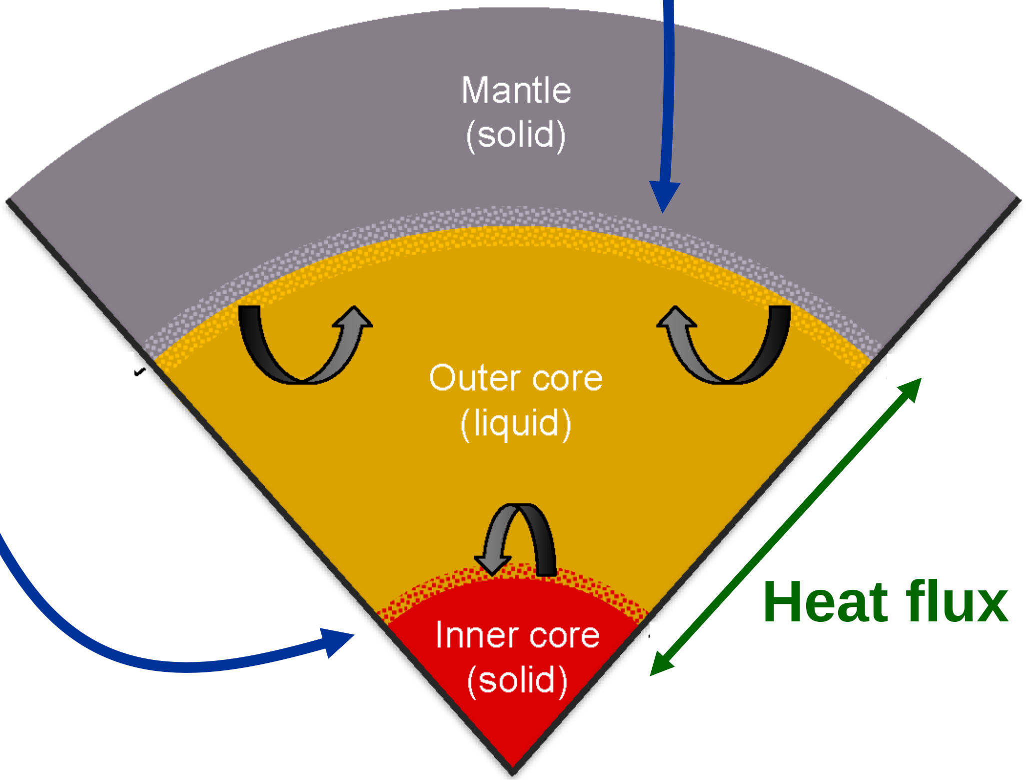
- **Rourke et al. (2017):** *"how to power convection in the core and thus a dynamo for the **vast majority of the Earth's history** remains one of **the most pressing puzzles in geophysics**"*

Two sources of buoyancy in planetary liquid cores

- **No inner core (e.g. Early Earth)**
  CMB exsolution *(Hirose et al., Nature, 2017)*



- **With an inner core**
  ICB crystallization

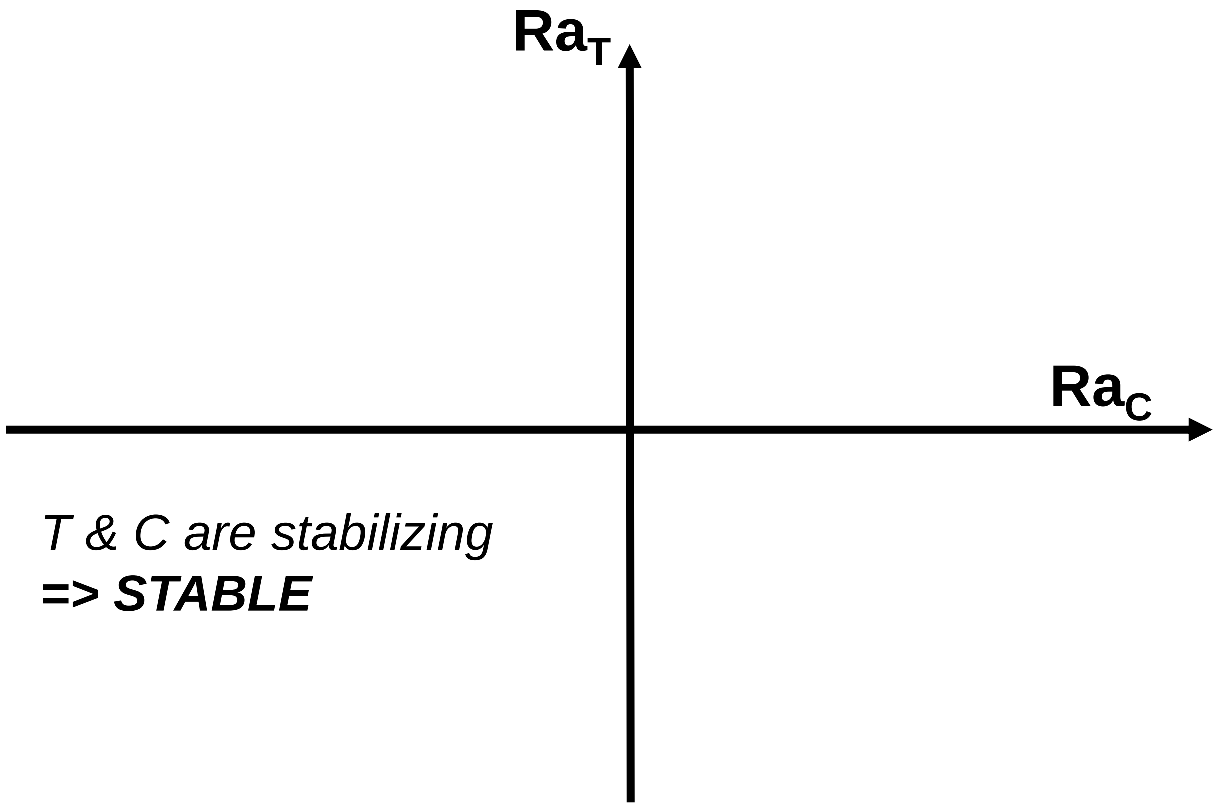


- **Stabilizing grad(T)**
- **Destabilizing grad(C)**

Double-diffusion in planetary liquid core

Dimensionless buoyancy

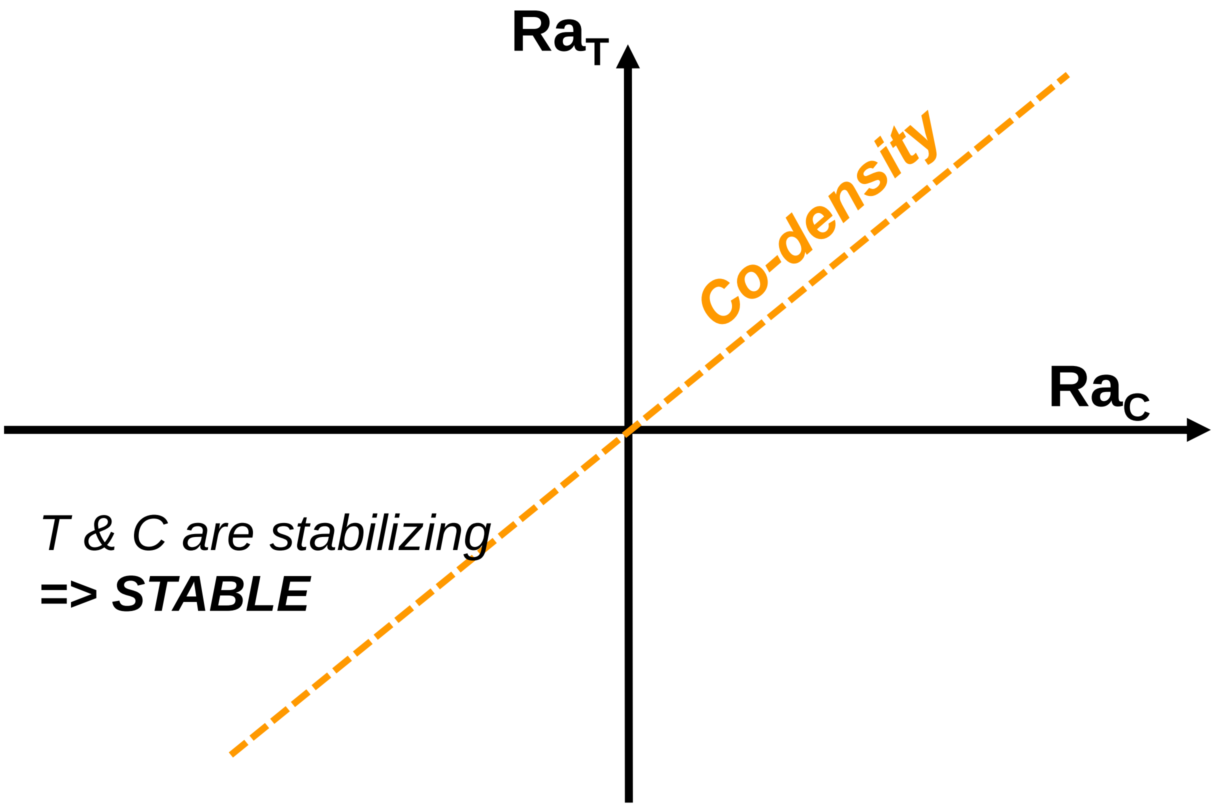
$$\delta\rho^* = Ra_T\ \delta T + Ra_C\ \delta C$$



Regimes & Onset

Double-diffusion in planetary liquid core

- **Usual** dynamo simulations**: co-density**



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Double-diffusion in planetary liquid core

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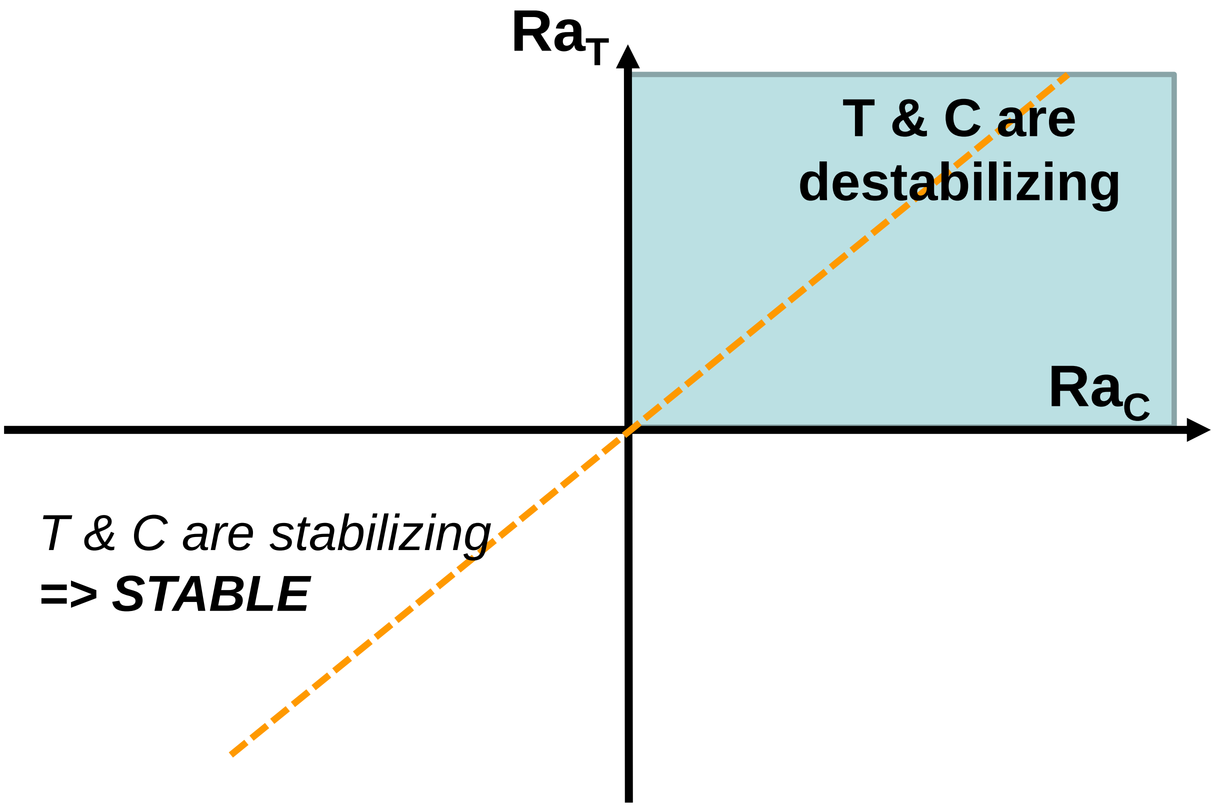

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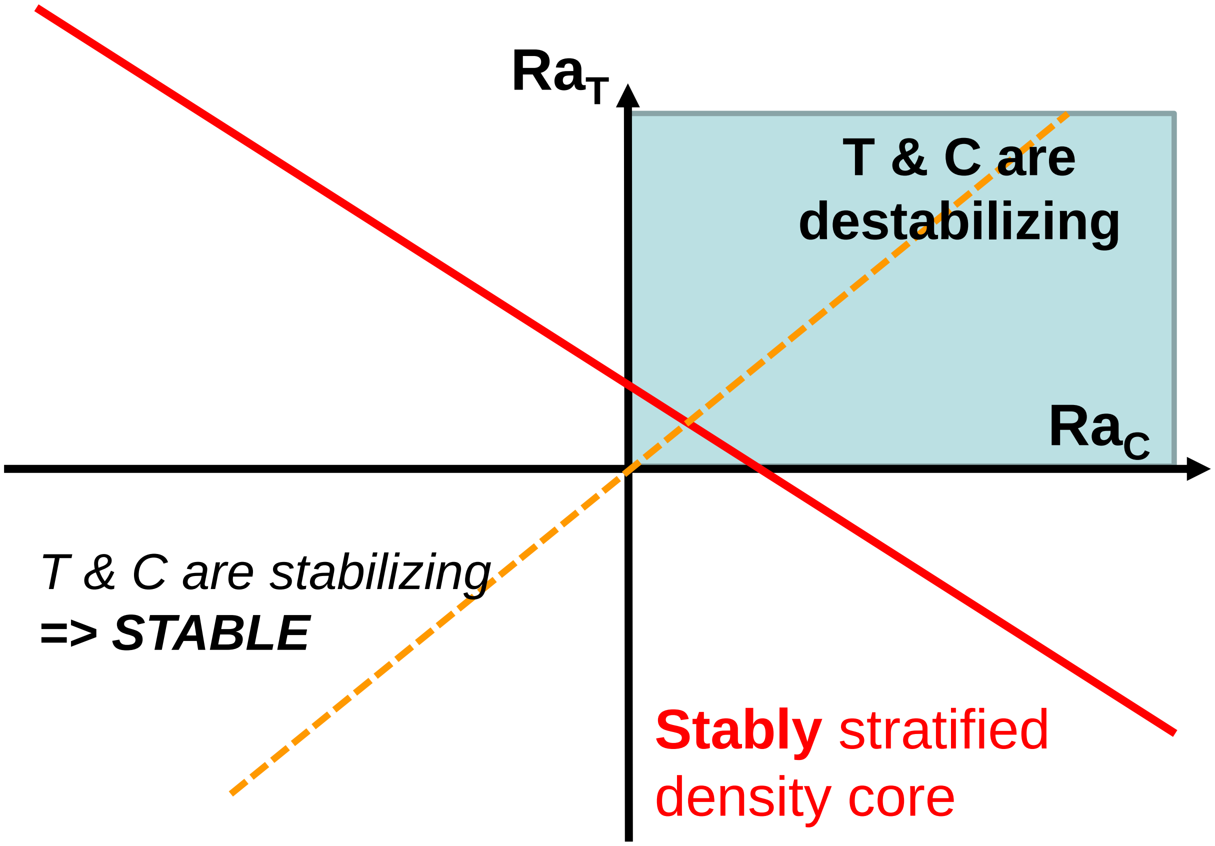
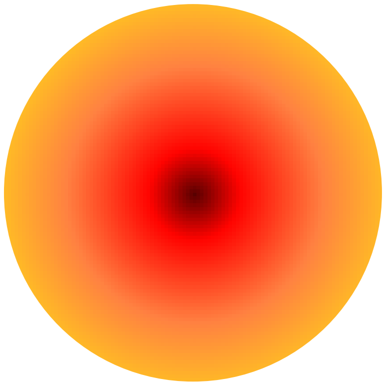


Regimes & Onset

Dimensionless buoyancy

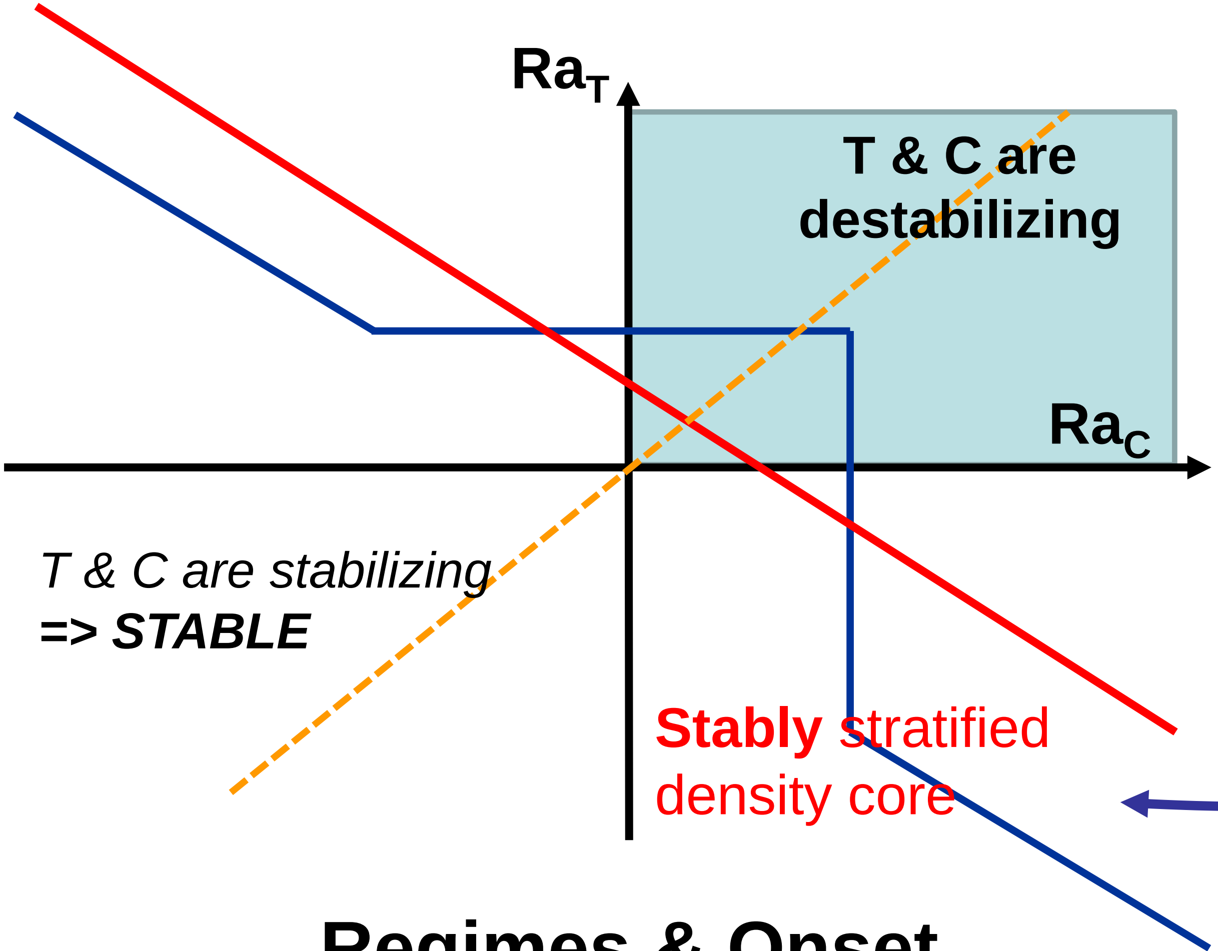
$$\delta\rho^* = Ra_T\,\delta T + Ra_C\,\delta C$$

No flow?



Double-diffusion in planetary liquid core

Usual approach: co-density



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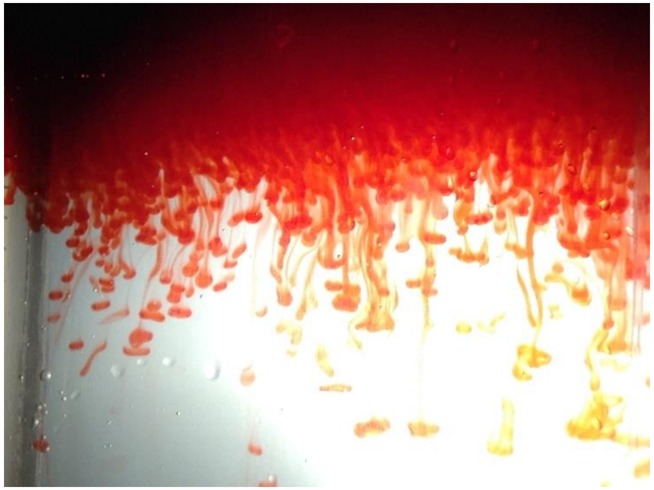
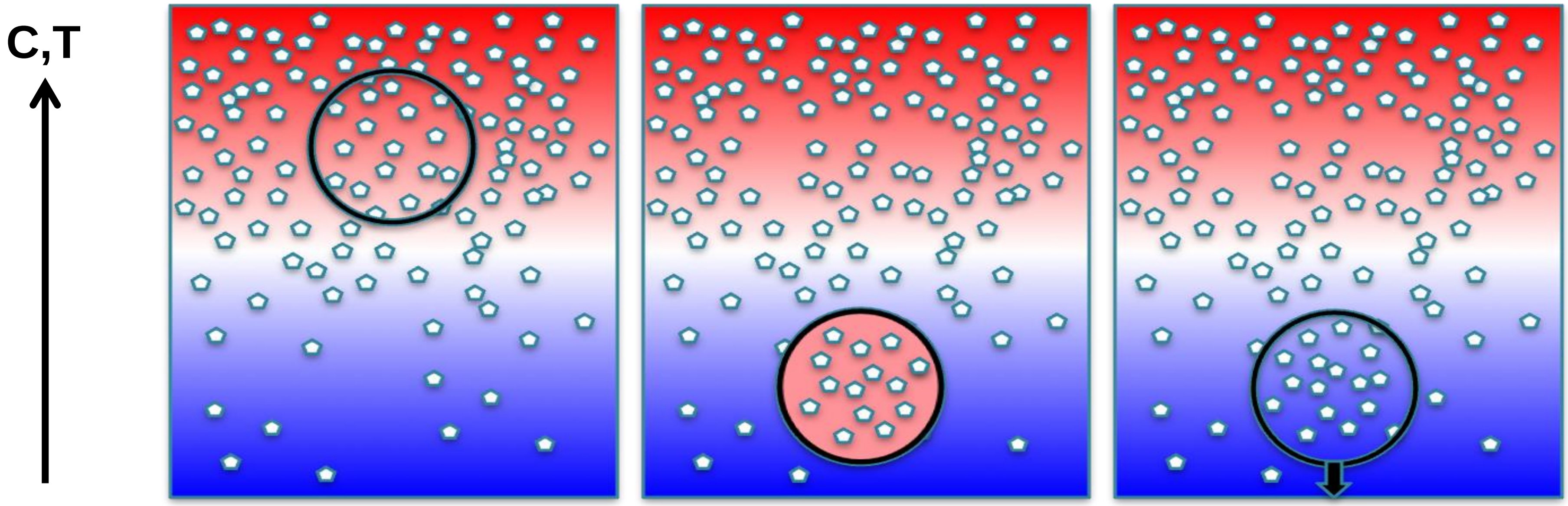


fingering

Regimes & Onset

Double diffusion onset (local stability analysis)

A little known double-diffusion in the Earth's core



Fast diffusion of T wrt. C

Instability well known in **ocean** & **stellar** physics

But almost no study in a **rotating spherical geometry**
(Manglik+10, Net+12, Bouffard's PhD)

Double diffusion in the Early Earth's core?

- **Scales:** R, R^2/ν, T, C, such that $T_0(r) = \frac{1-r^2}{Pr}, \quad C_0(r) = \frac{1-r^2}{Sc}$

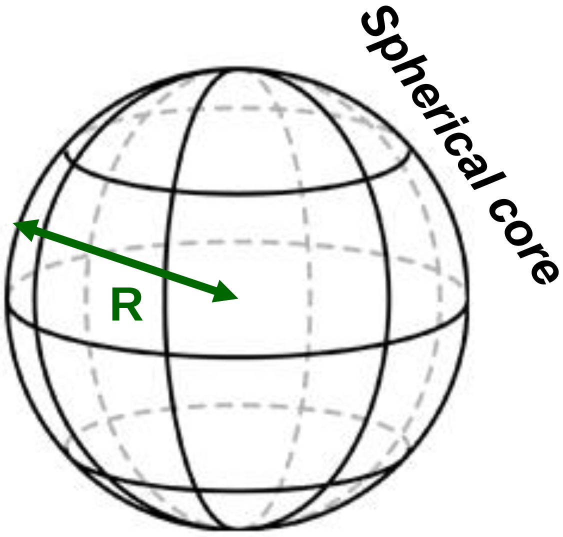
- **Equations**

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\frac{2}{Ek}\mathbf{1}_z \times \boldsymbol{u} - \nabla p + \boldsymbol{\nabla}^2\boldsymbol{u} + (Ra_T\,\Theta + Ra_C\,\xi)\; r\mathbf{1}_r,$$

$$\frac{\partial \Theta}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\Theta = \frac{1}{Pr}\left(2\,\boldsymbol{r}\cdot\boldsymbol{u} + \nabla^2\Theta\right),$$

$$\frac{\partial \xi}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\xi = \frac{1}{Sc}\left(2\,\boldsymbol{r}\cdot\boldsymbol{u} + \nabla^2\xi\right),$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0,$$



$$Ra_T = \frac{\alpha_T g_0 Q_T R^6}{6\nu\kappa_T{}^2}, \quad Ra_C = \frac{\alpha_C g_0 Q_C R^6}{6\nu\kappa_C{}^2} \qquad Ek = \frac{\nu}{\Omega_s R^2}, \quad Pr = \frac{\nu}{\kappa_T}, \quad Sc = \frac{\nu}{\kappa_C}$$

Planetary cores: **$<10^{-10}$** **$L=Sc/Pr>10^3$**

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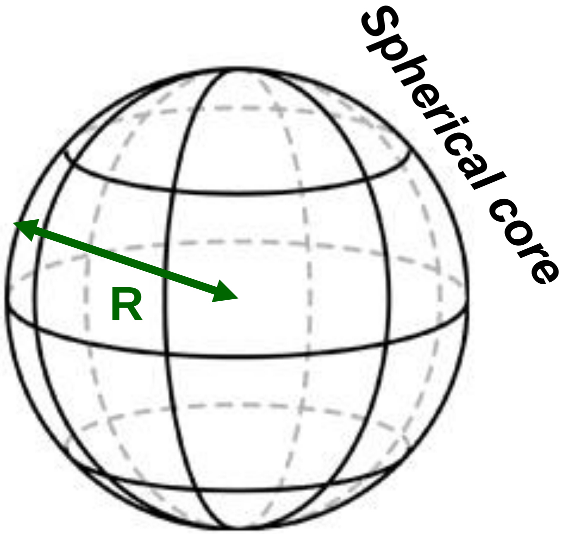

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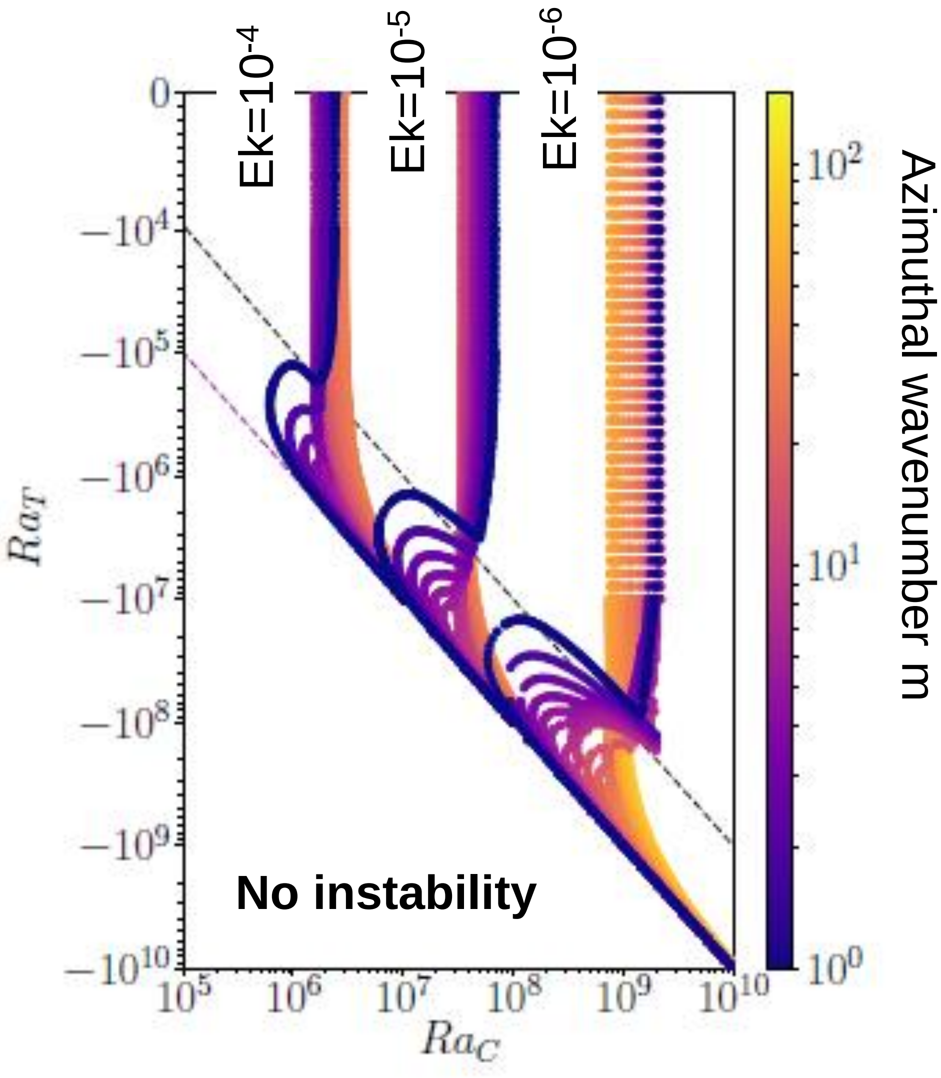
- **Codes:** open-source, pseudo-spectral => **SINGE** (linear, by J. Vidal) **& XSHELLS** (non-linear, by N. Schaeffer)

Onset in a rotating sphere at L=Sc/Pr=10

Earth

Ek=10^{-15}



Code SINGE

Monville et al., GJI, subm.

Onset in a rotating sphere at L=Sc/Pr=10

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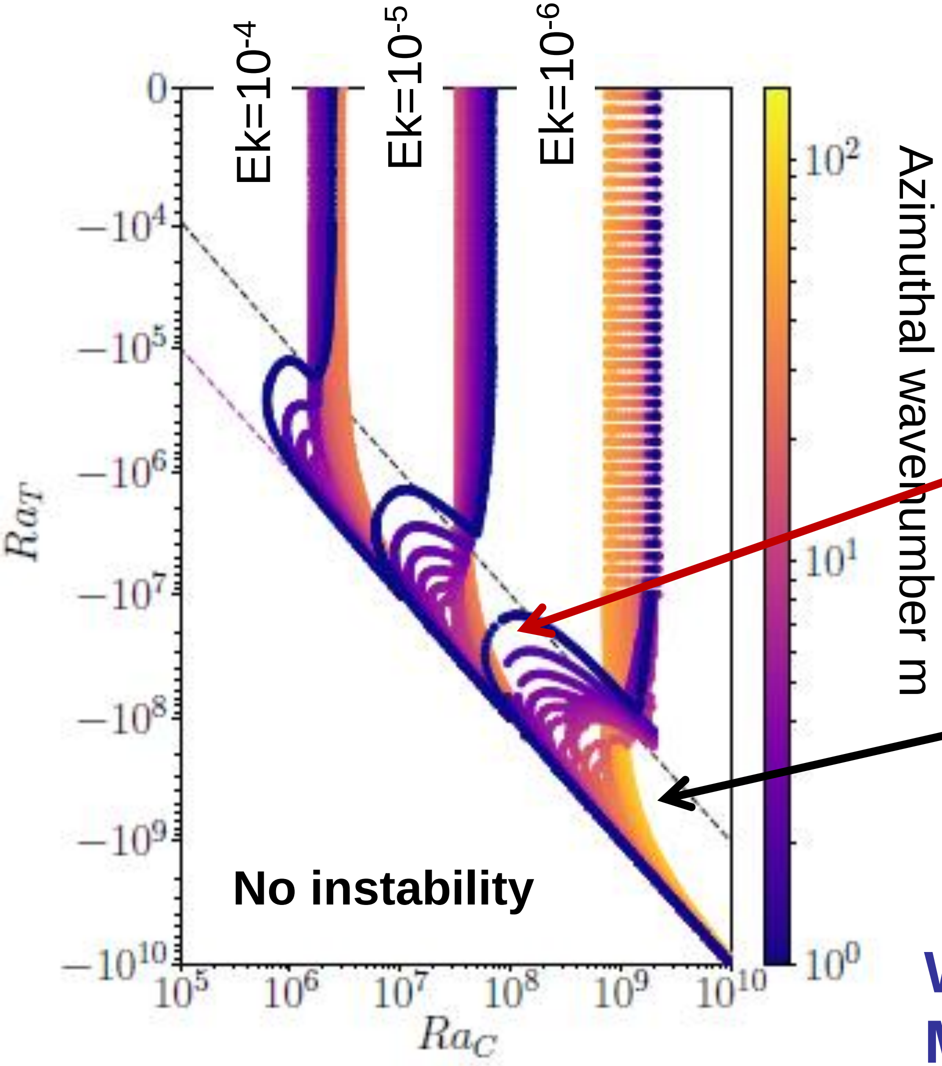


Code SINGE

Onset in RaC is decreased by a stabilizing thermal gradient!

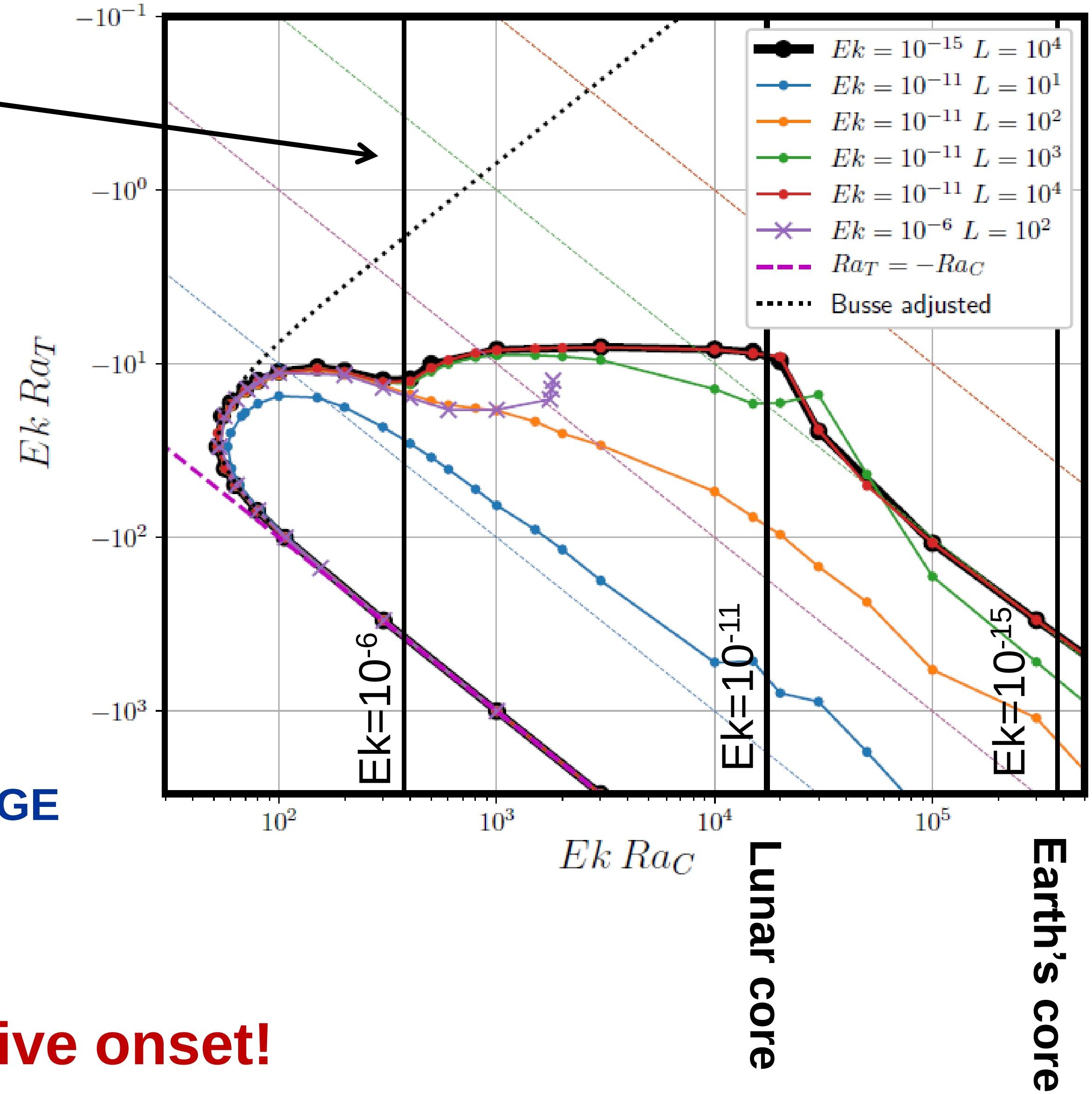
Stably stratified fluid

What about the drop for Earth & Moon's cores at L>10^3 ?

Onset in a rotating sphere at core values

L=Sc/Pr

<u>Convection onset</u>
(for $Ra_T=0$)



Code SINGE

Inviscid double diffusive onset!

Inviscid convection domain

L=Sc/Pr

Thus, DDC onset varies as **Ek^{-1}**

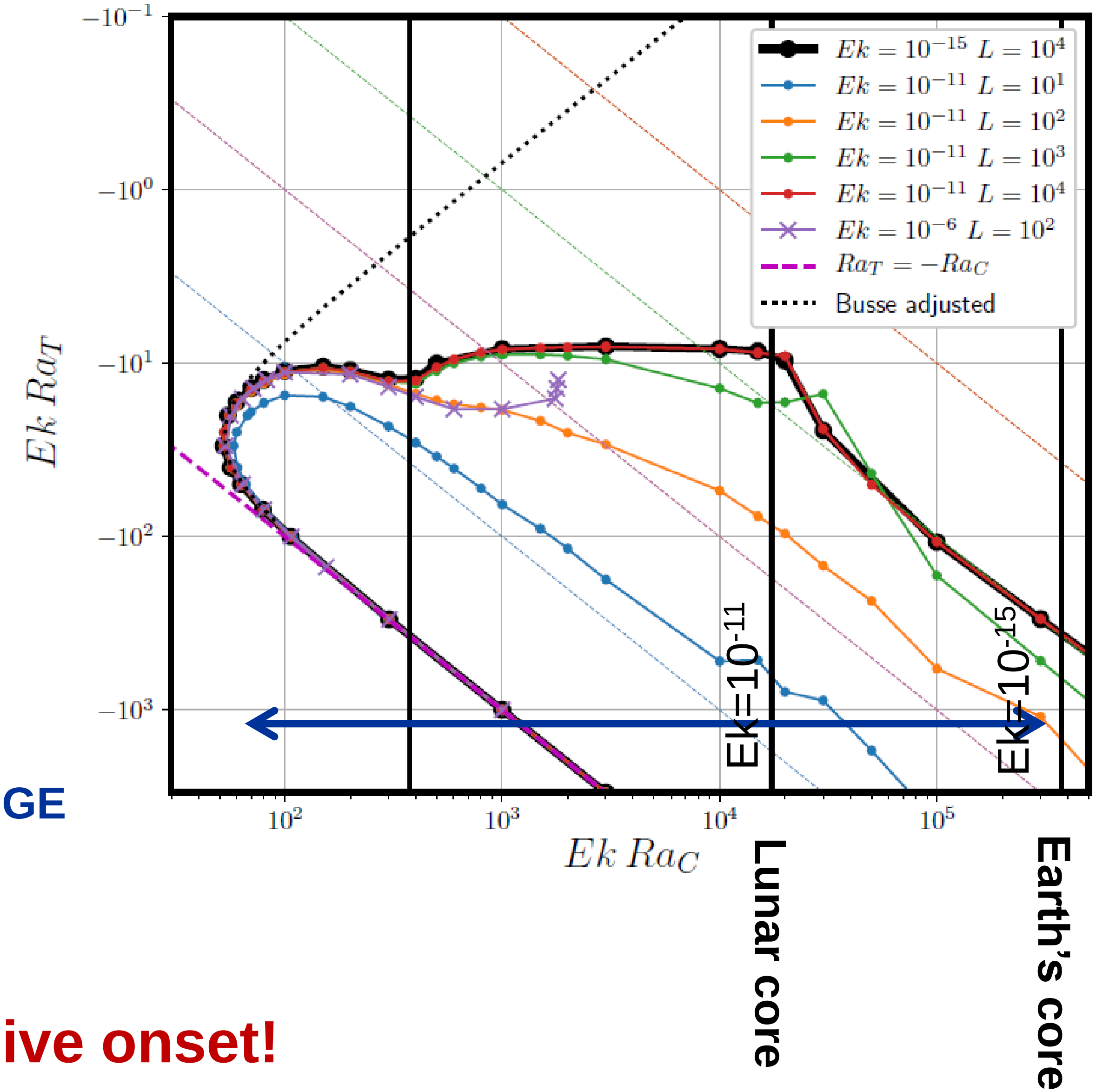
Since usual convection onset varies as **$Ek^{-4/3}$**

⇒ Onset drop ~ $Ek^{-1/3}$

⇒ Earth: drop of 10^4

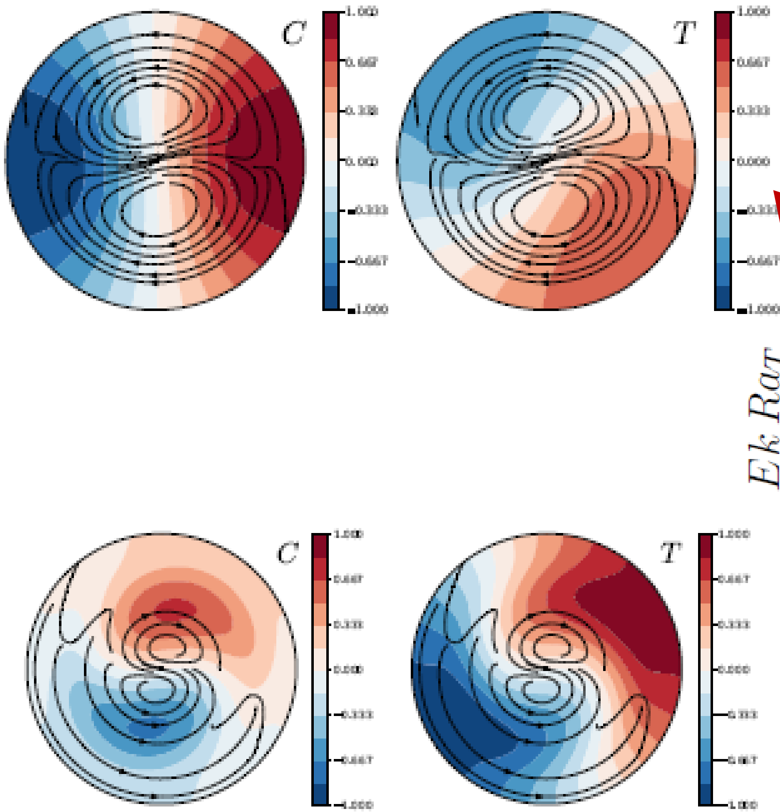
⇒ Moon: drop of 10^3

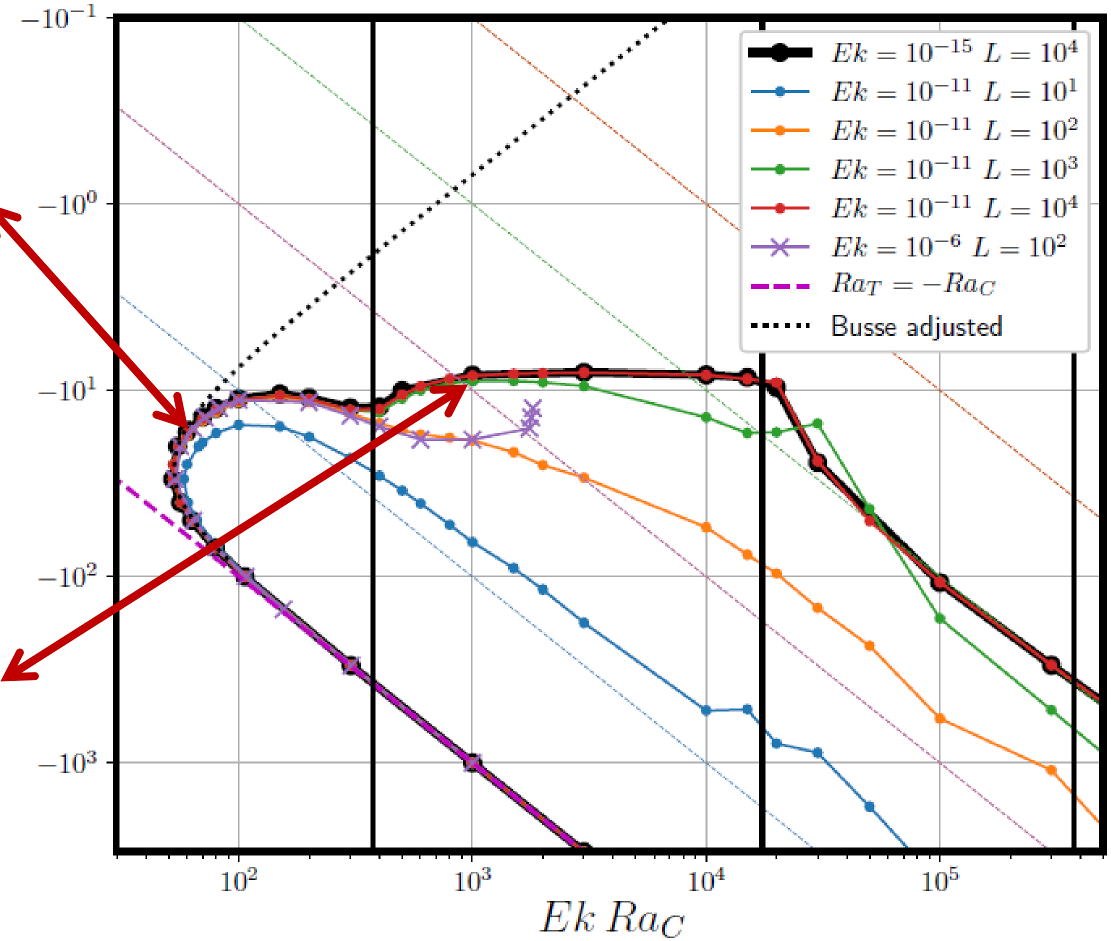
Code SINGE



Inviscid double diffusive onset!

Eigenmodes at the onset





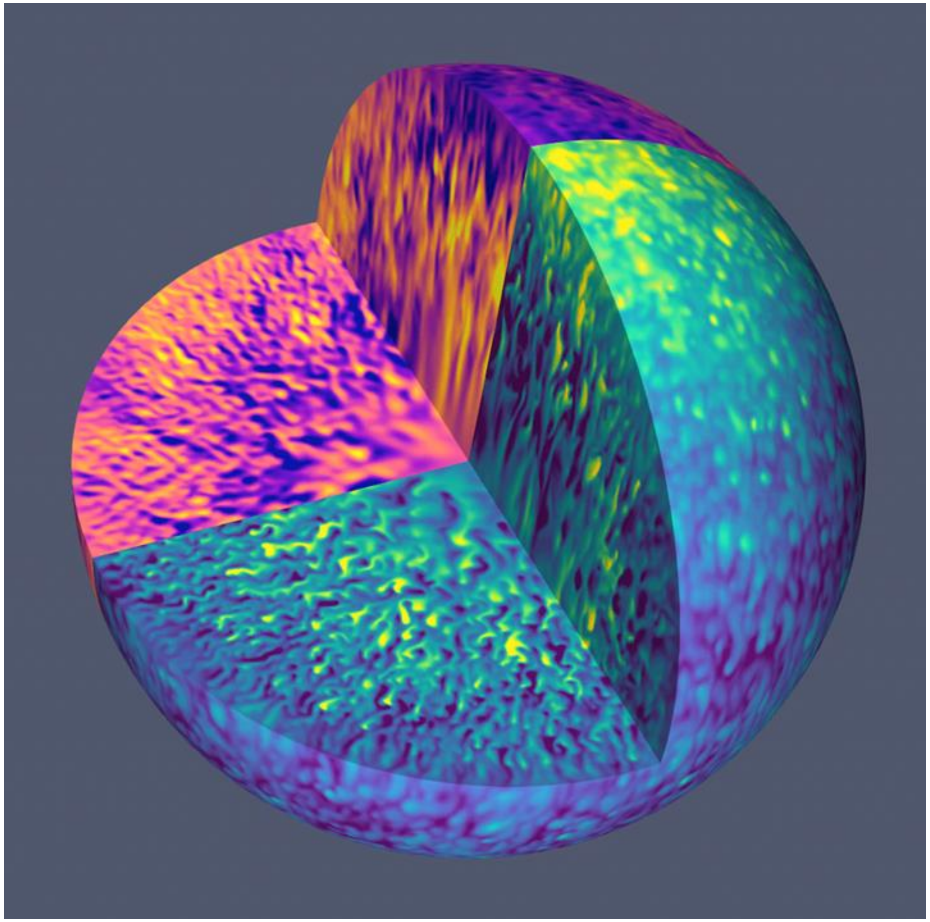
Large-scale inviscid double diffusive flow in the fingering regime

Non-linear regime?

Non-linear rotating double diffusive convection

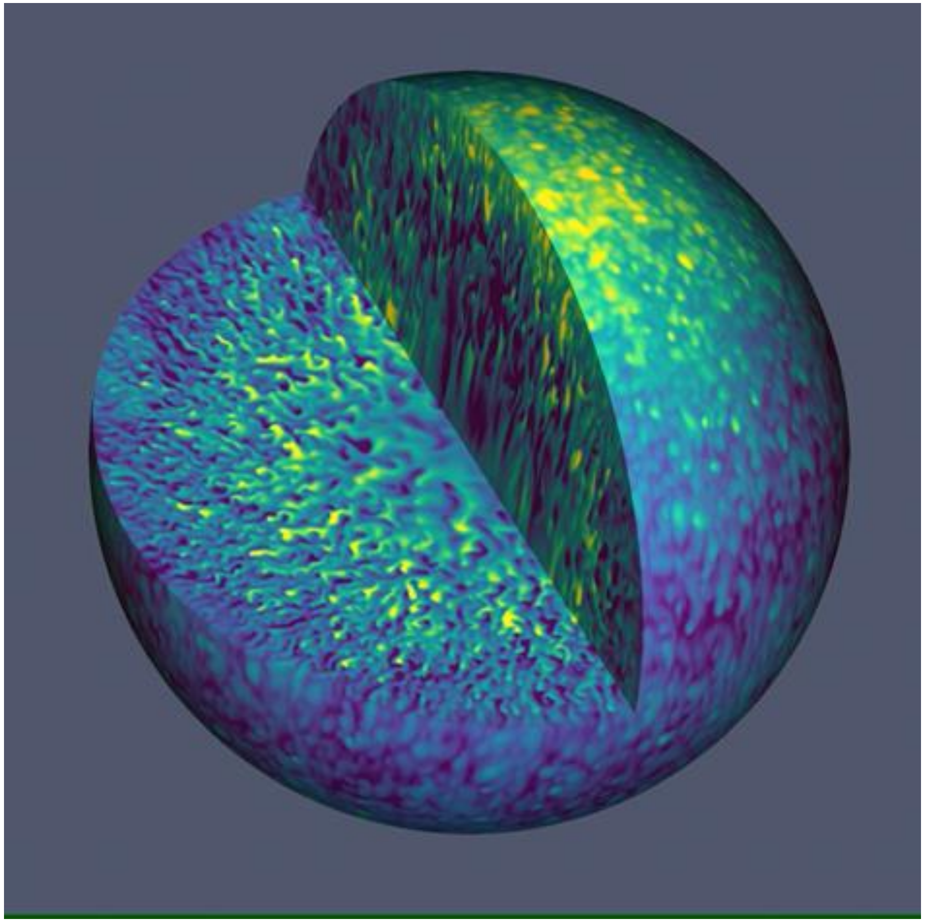
Thermal (upper) & solutal (lower) buoyancy

Video

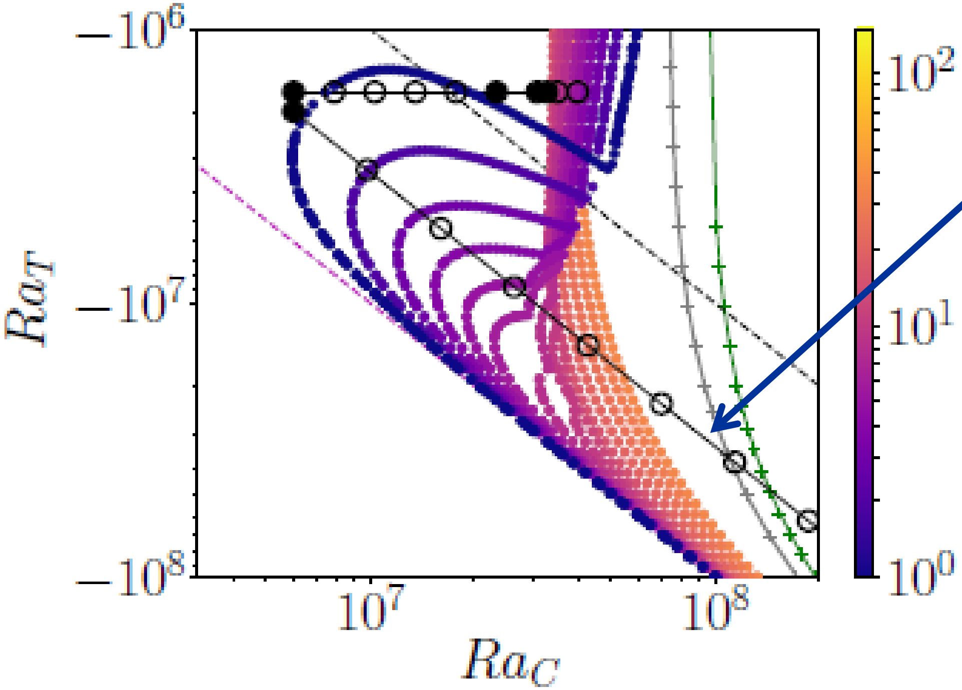


Vorticity magnitude

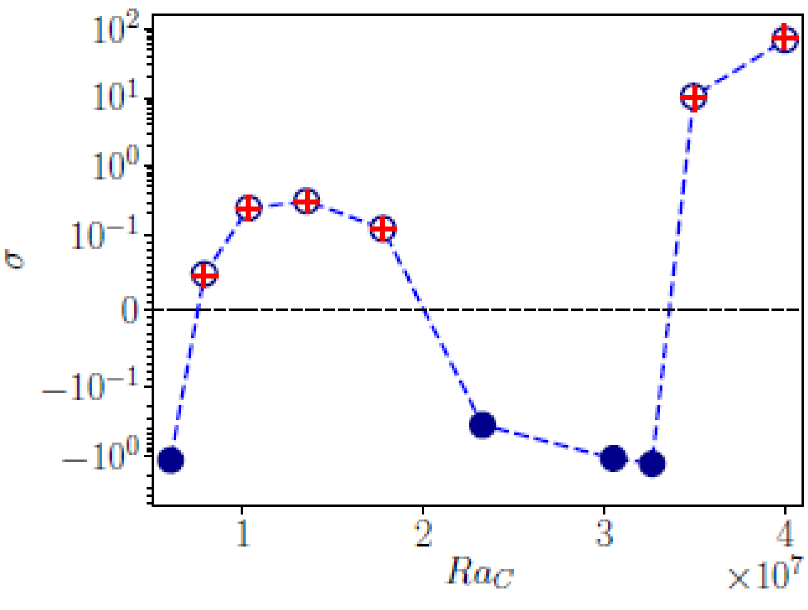
Video



Double diffusive growth rates at L=Sc/Pr=10

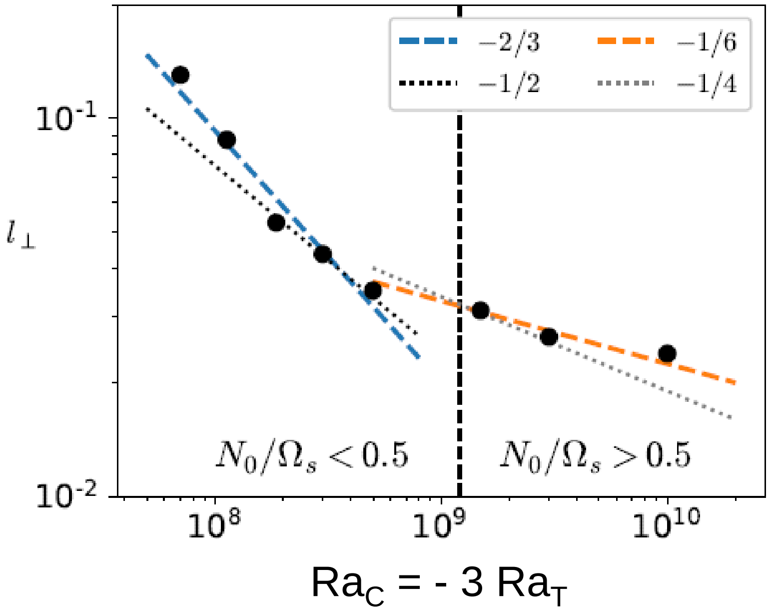


Stay on this line, at constant distance from neutral buoyancy



At constant Ra_T , the onset is not uniquely defined….

Double diffusive structures & transport at L=Sc/Pr=10

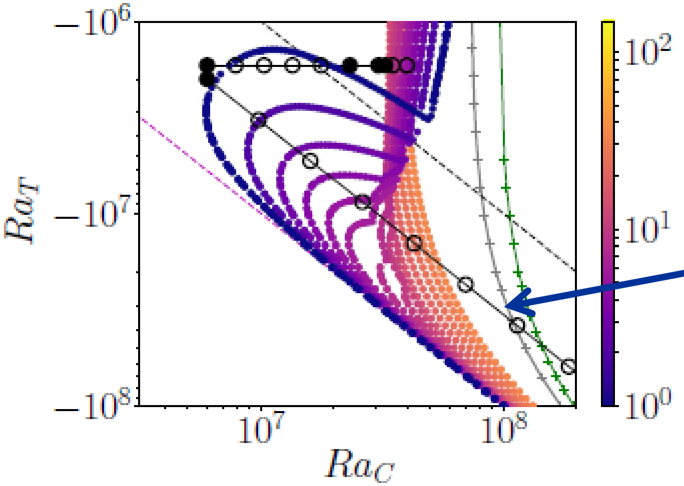


Radko, 2013, **non-rotating** : -1/4

Bouffard, 2017, **rotating**: -1/2

=> Transition between 2 regimes

(weakly and strongly stratified)



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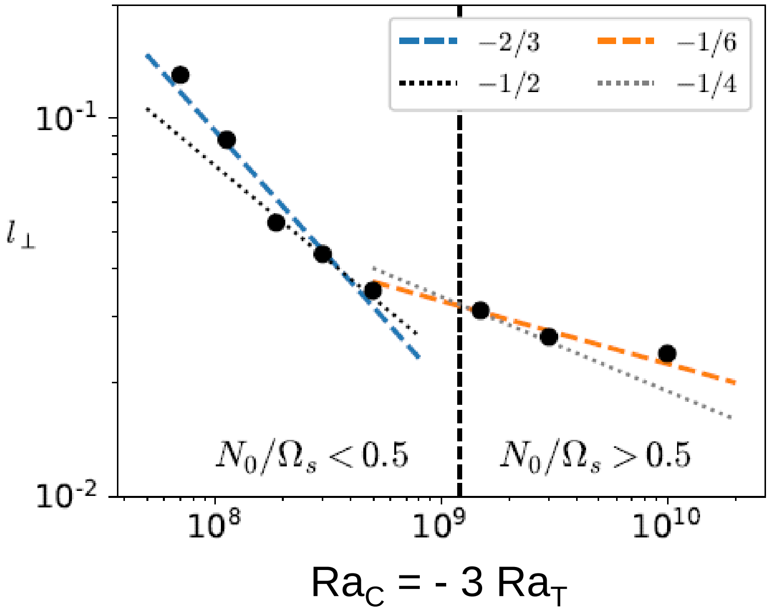
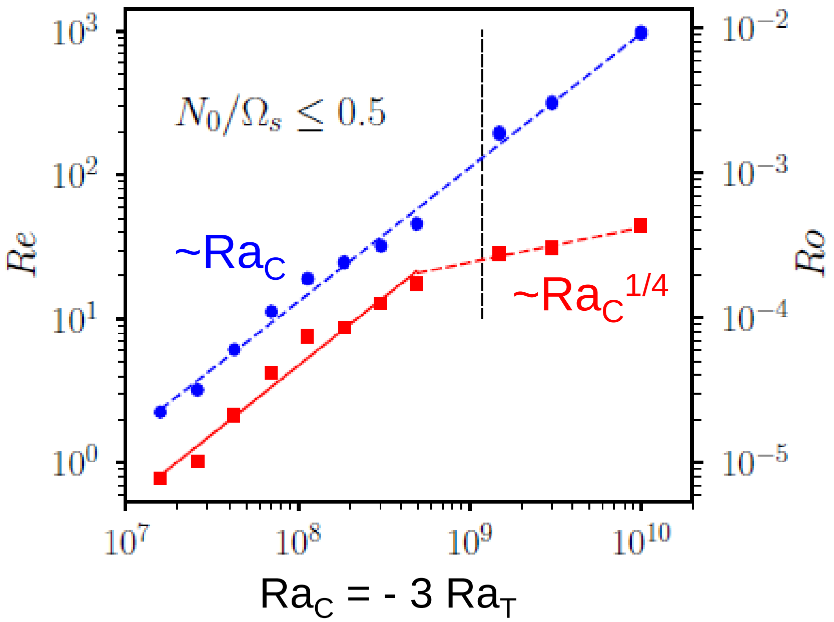
Double diffusive structures & transport at L=Sc/Pr=10



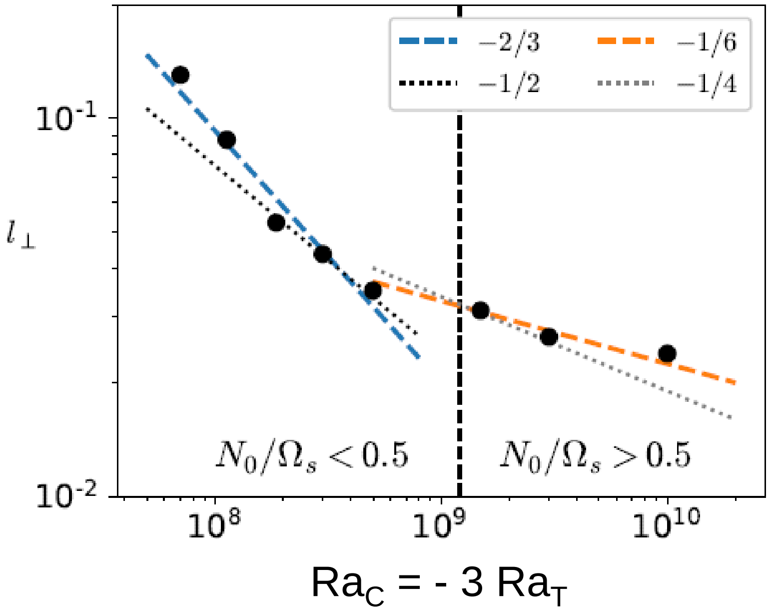


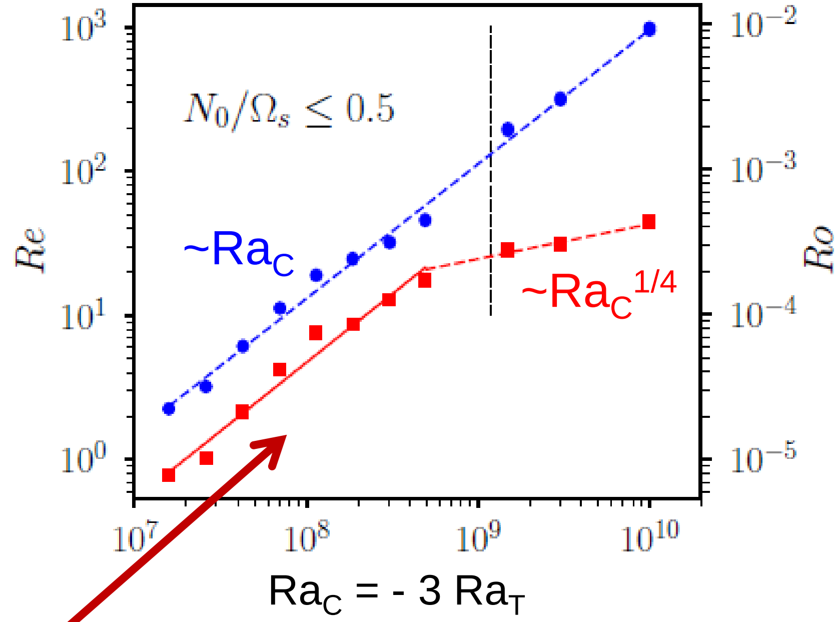
Re based on

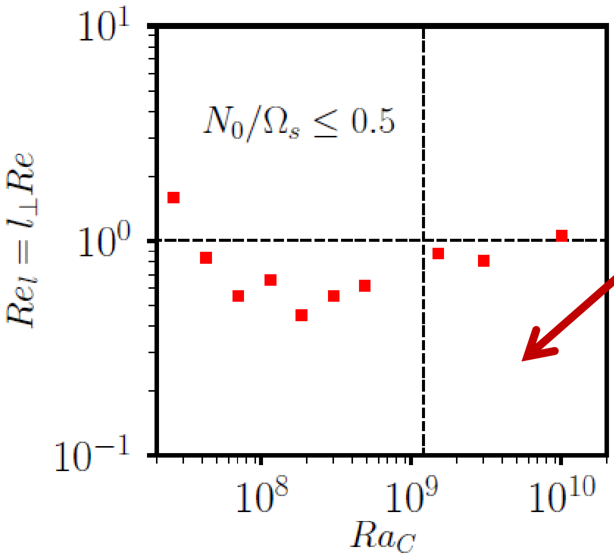
- the total NRJ (blue)
- the poloidal non-zonal NRJ (red) = proxy of the radial velocity

Monville et al., GJI, subm.

Double diffusive structures & transport at L=Sc/Pr=10







Re based on

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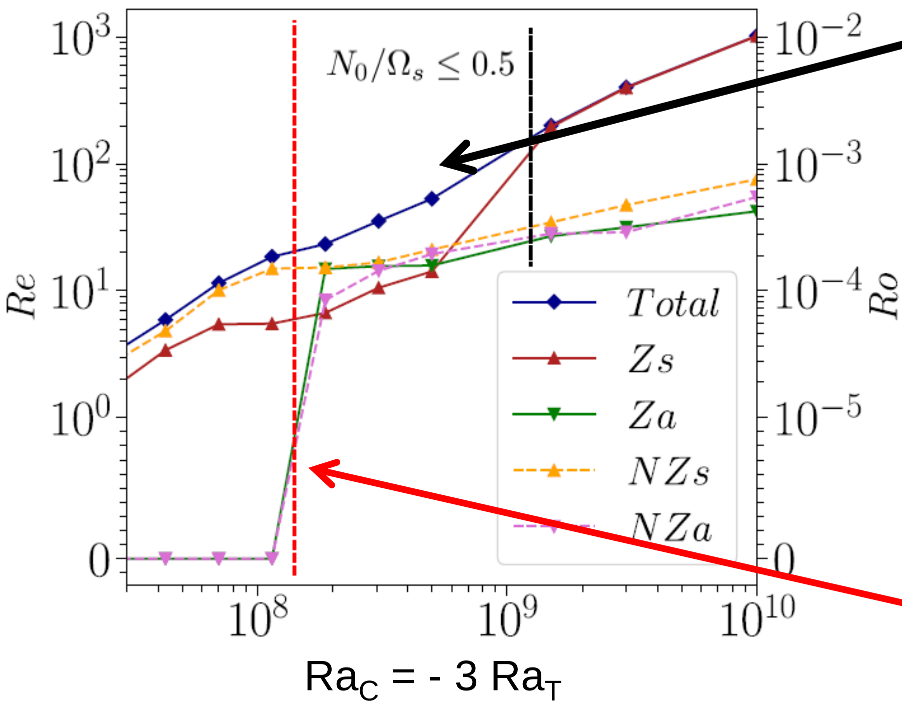
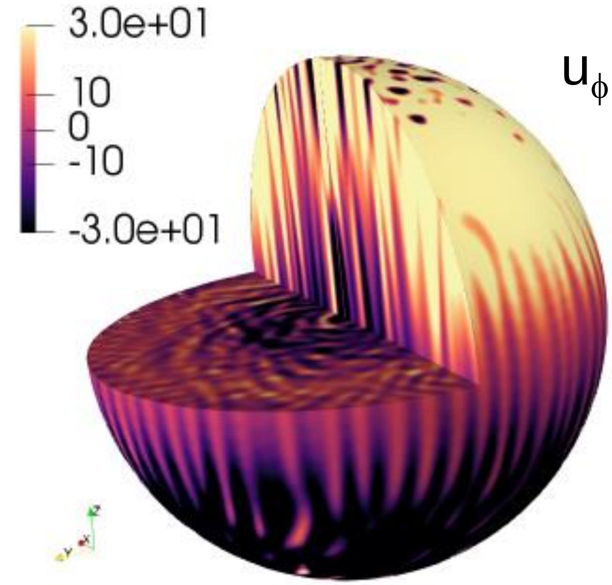
Such that $Re_l = [Pr(R_0-1)]^{-1/2} \sim 1$
(Garaud 2018)

Monville et al., GJI, subm.

Zonal flows

$Pr = 0.3, Sc = 3, Ek = 10^{-5}$



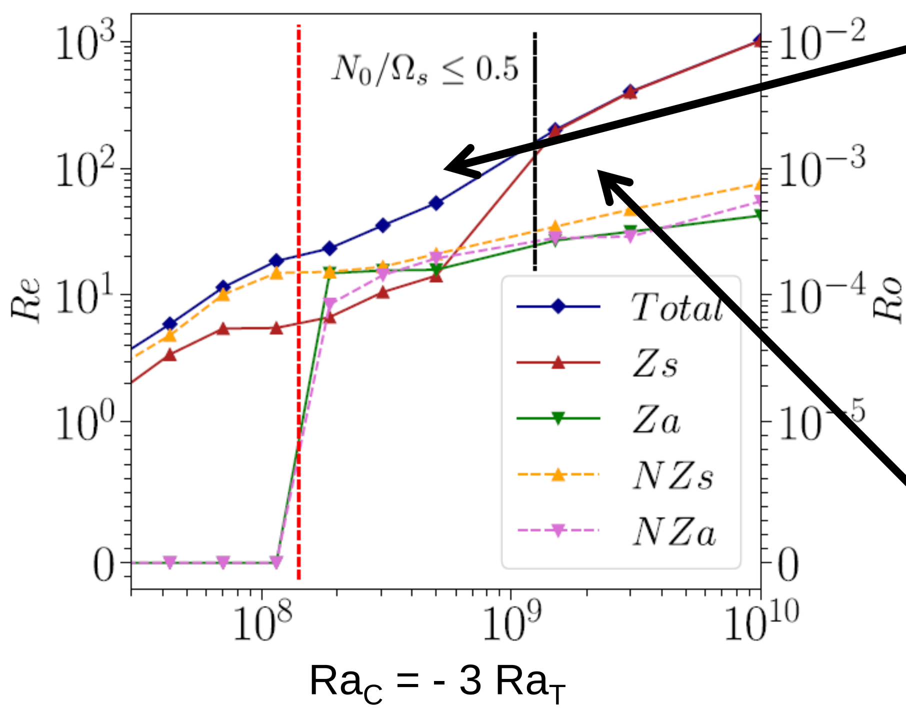


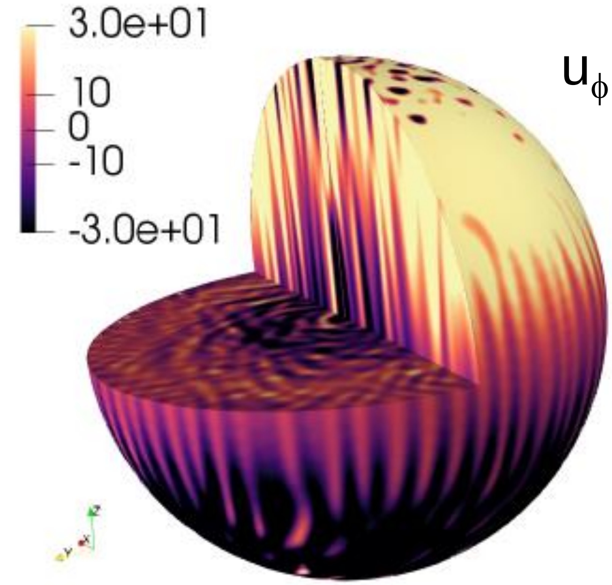
Anti-symmetric linear onset for the mode m=0

- Far from RDDC onset, **equatorially anti-symmetric zonal flow**
- Well predicted by a purely linear mechanism

Zonal flows

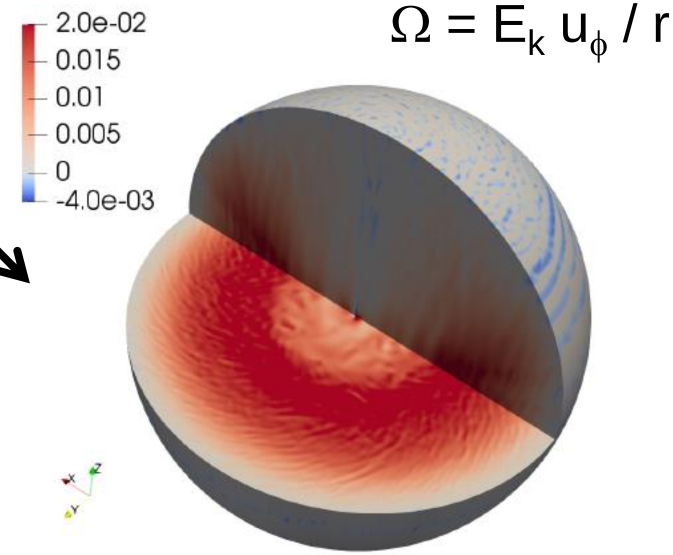
$Pr = 0.3, Sc = 3, Ek = 10^{-5}$





u_ϕ

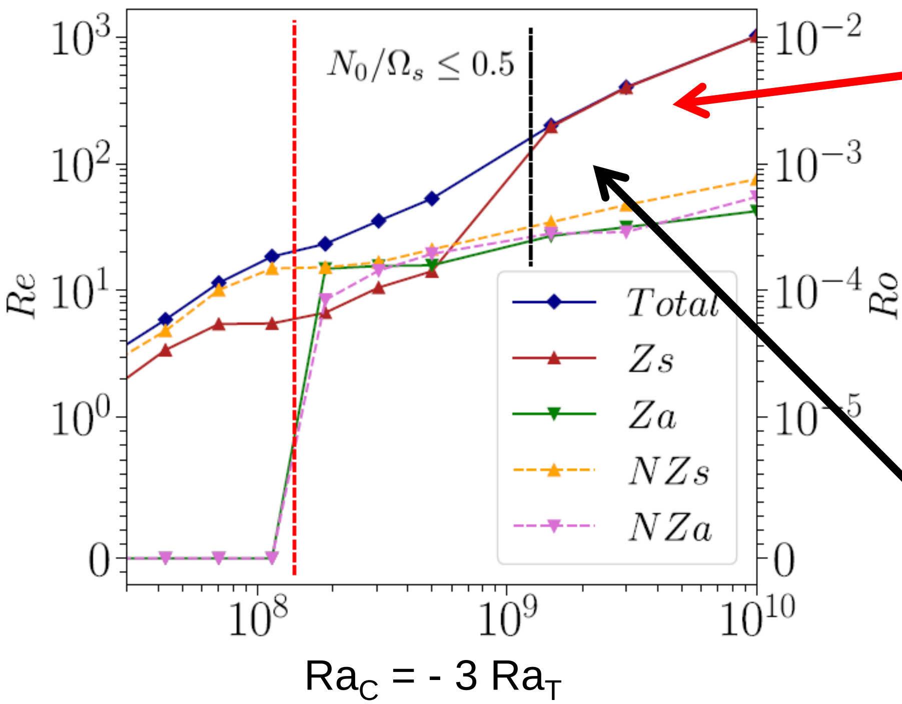
$\Omega = E_k\, u_\phi / r$



For stronger stratification, **equatorially symmetric intense zonal flows**

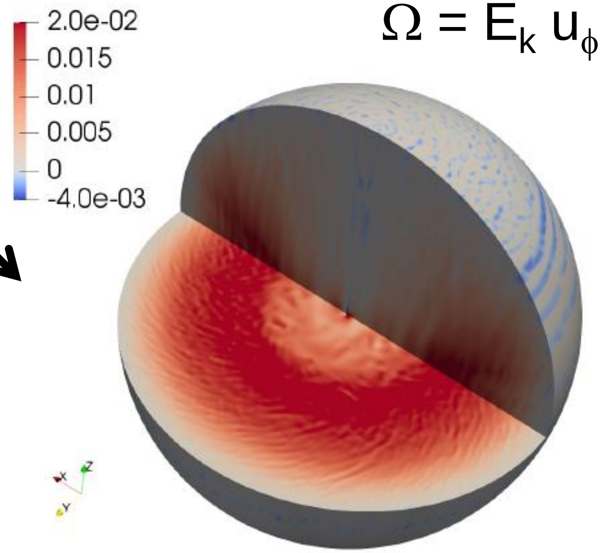
Zonal flows

$Pr = 0.3, Sc = 3, Ek = 10^{-5}$



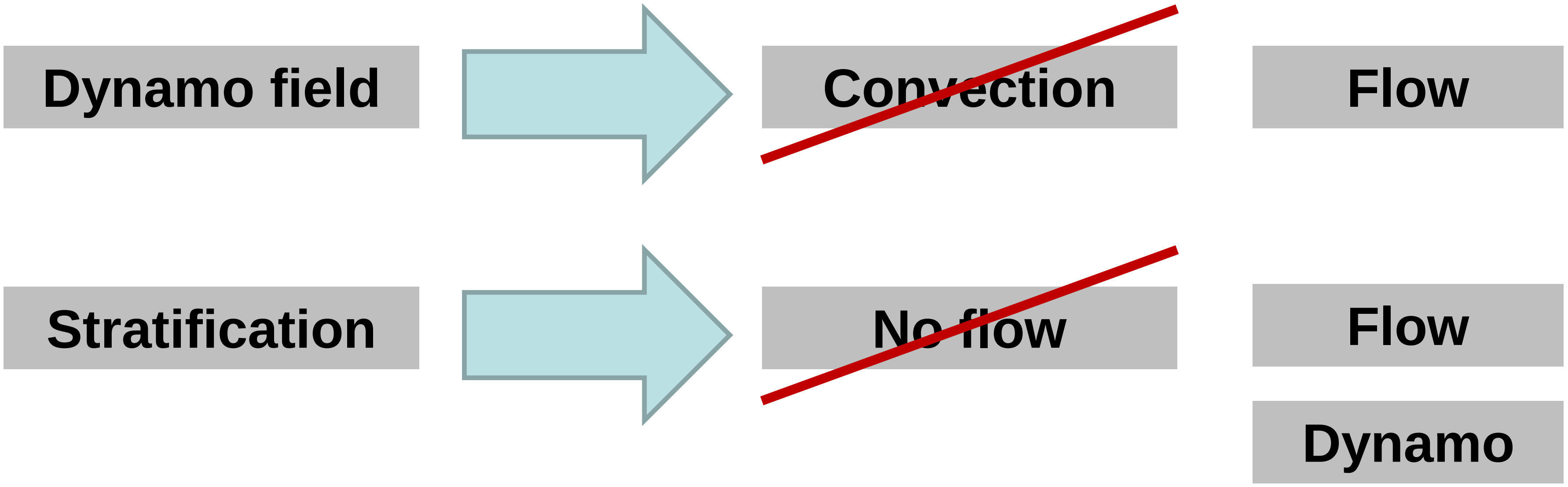
Current work: double-diffusive **dynamos** in stably stratified fluids?

$\Omega = E_k\, u_\phi / r$



For stronger stratification, **equatorially symmetric intense zonal flows!**

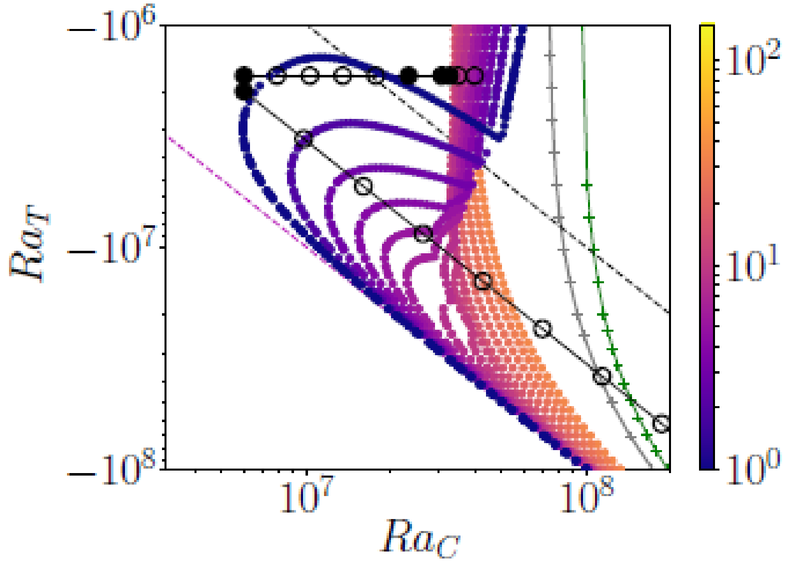
Conclusions



The same for mechanical dynamos…

More details: Monville et al. (2019) arXiv:1902.08523
Code: https://nschaeff.bitbucket.io/xshells/

Double diffusive transport



$$Nu_T = \frac{T_0(0) - T_0(1)}{T_0(0) - T_0(1) + \Theta_{rms}(0) - \Theta_{rms}(1)},$$

$$Sh = \frac{C_0(0) - C_0(1)}{C_0(0) - C_0(1) + \xi_{rms}(0) - \xi_{rms}(1)},$$