Investigations on precession, tides, convection and stratification in planetary cores

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Outline: exploring "emerging" physics of planetary cores

Precession and Tides have been advocated to be able to drive dynamos in planetary cores. Meanwhile, stably stratified layers at the top of the core have been proposed. Here, I will review a few results obtained with collaborators in the last years on related topics.

1. Part 1: Some effects of stratification
   - Rossby waves VS stratification
   - Convective dynamo VS stratification
   - Tides VS stratification
   - Double-diffusive convection VS stratification

2. Part 2: The effects of precession on planetary cores
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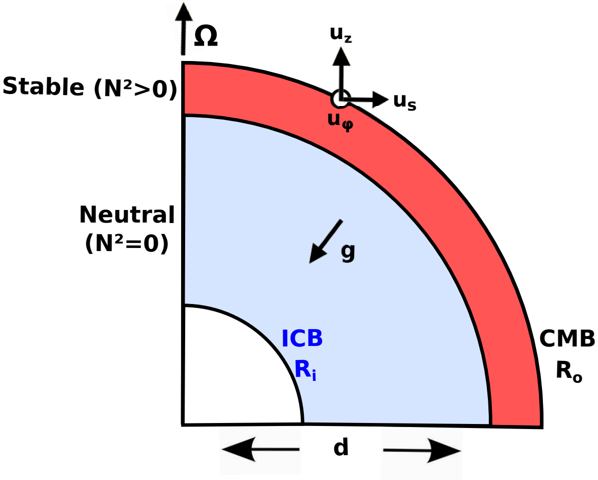
Rossby waves VS stratification

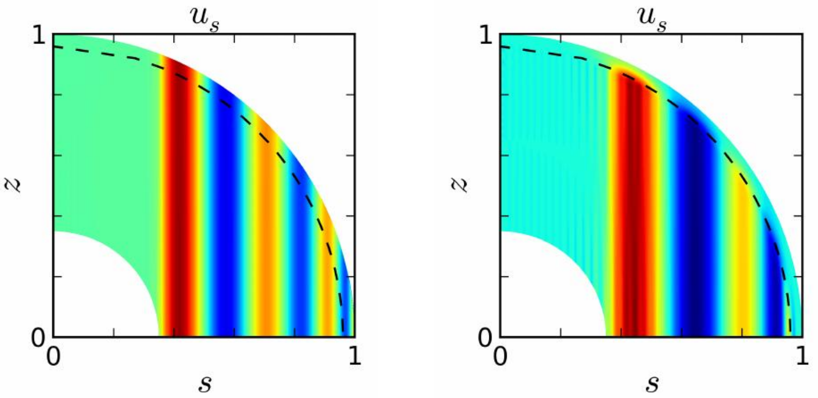
How are Rossby waves in the Earth's core affected by a stratified layer?

The model: a thin stratified layer



Question: how does such a layer affects waves in the core?

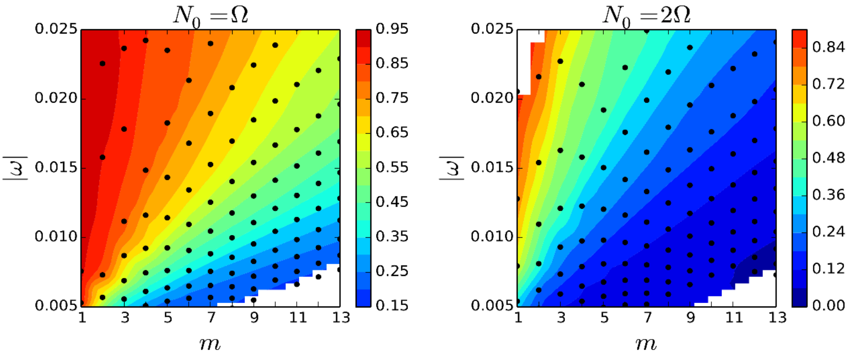
Method: compute quasi-geostrophic eigenmodes



weak stratification ($N_0 = 0.1\,\Omega$) strong stratification ($N_0 = 2\,\Omega$).

Vidal & Schaeffer GJI 2015

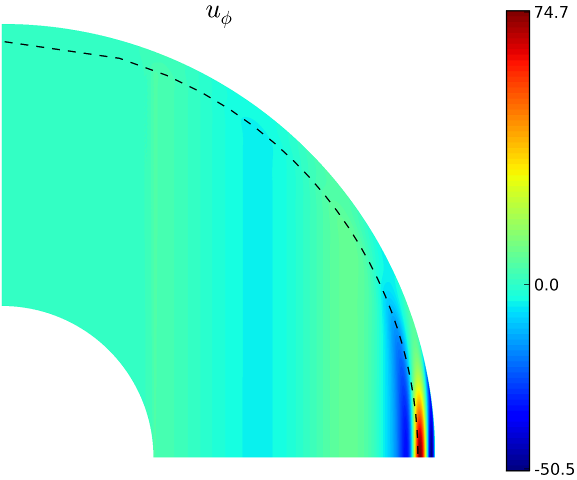
Result: Fast and large-scale modes shall pass



In agreement with Takehiro & Lister (2001, 2002), Nakagawa (2011) and Gastine+(2020) : the larger, the easier to pass.

Vidal & Schaeffer GJI 2015

Equatorial trapping



Some modes exist that concentrate near the equator (Friedlander+ 1982, Crossley 1984)

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Convective dynamo VS stratification

- **How is the dynamo affected by a stratified layer?**
- **Can the geodynamo put constraints on the stratified layer?**

Result: ?

The simulations of Gastine, Aubert & Fournier (GJI 2020):

- agree with the penetration depth of Takehiro & Lister (2001): the larger, the easier to pass.
- convection near the CMB happens mostly at very small scale (viscous scale) in the simulations

Extrapolation to the core: no large and strong stratified layer is compatible with today's magnetic field.

Result: ?

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Extrapolation to the core: no large and strong stratified layer is compatible with today's magnetic field.

- **However, the turbulent convective length-scale is much larger than the viscous one ! (see Guervilly+ Nature 2019)**.
- Larger convective scales (x300 to x1000) may well be enough to pass a rather deep ($\simeq 100$km) and strongly stratified layer ($N/\Omega \simeq 1$).

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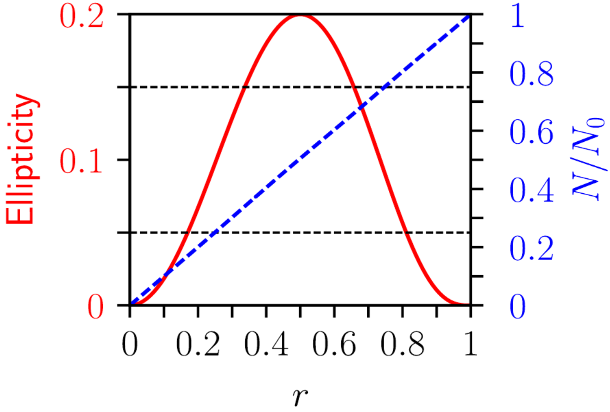
Tides VS stratification

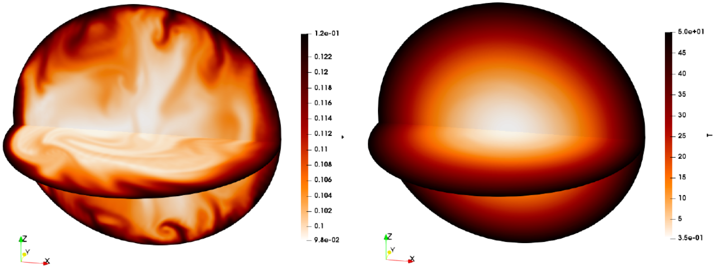
- Can tidally-driven flow overcome stratification?
- May they still drive a dynamo?

Method: direct numerical simulations



Artificial base state that mimics the effect of tides on an elliptic container (Vidal+ MNRAS 2018)

Results: ubiquitous elliptic instability; erosion for $N/\Omega < 1$



$N_0/\Omega_s = 0.5$ $N_0/\Omega_s = 10$

- The elliptical instability develops for $N/\Omega < 1$ (expected)
- The elliptical instability develops for $N/\Omega > 1$ (rather unexpected)
- Radial mixing (and stratification erosion) only occurs for $N/\Omega < 1$, mostly horizontal motions otherwise.

Vidal+ MNRAS 2018

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Motivation: Geodynamo without an inner-core?

- Today, most convective power is thought to come from light-element release at the inner-core boundary due to inner-core crystallization.

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Question:

With a subadiabatic thermal gradient and unstable composition gradient, double-diffusive effects may play a role.
What do we know about double-diffusive convection in planetary interiors?

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Question:

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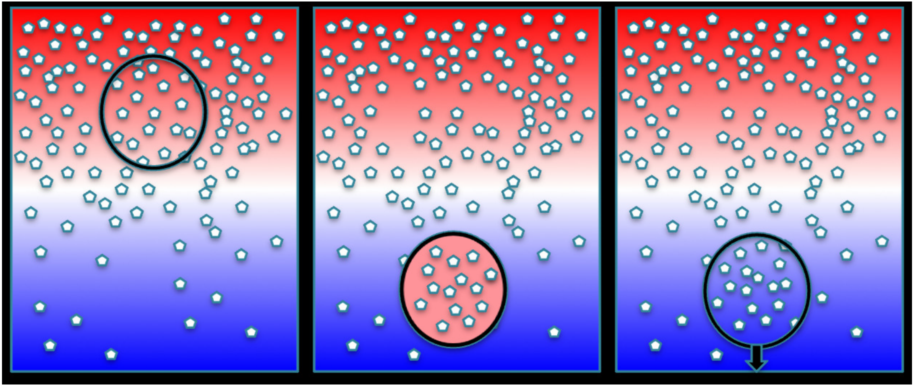
What do we know about double-diffusive convection in planetary interiors?

Not much ...

Double-diffusive convection: Finger convection



Well-studied in the context of oceanography.

See also Hage&Tilgner (2010) and Kellner&Tilgner (2014) for surprising new experimental results.

Rotating double-diffusive convection

- Busse (2002): claims the onset can be reduced to the one of non-rotating convection... (spoiler: he was wrong).

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- Simitev (2011): numerical onset in the anulus: lower onset for stably-stratified fluid.
- Net+ (2012): numerical onset in a sphere with inner-core: similar effect documented.

Numerical onset with SINGE: global survey

SINGE uses an eigensolver to compute modes in spherical geometry. Written by J. Vidal (see Vidal+ 2015).

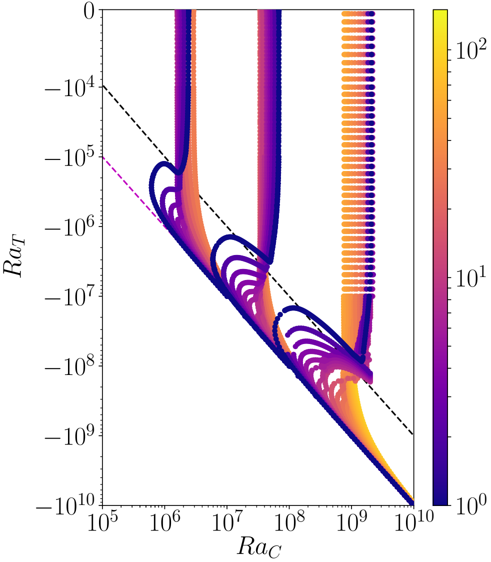
- Full-sphere (no inner-core)
- $E = 10^{-4}$, $E = 10^{-5}$, $E = 10^{-6}$
- focus on the *finger regime* – stable thermal gradient ($Ra_T < 0$), unstable composition gradient ($Ra_C > 0$).

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} &= -\frac{2}{Ek} 1_z \times \boldsymbol{u} - \nabla p + \boldsymbol{\nabla}^2 \boldsymbol{u} + (Ra_T\, \Theta + Ra_C\, \xi)\; r 1_r, \\
\frac{\partial \Theta}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\Theta &= \frac{1}{Pr} \left(2\, \boldsymbol{r} \cdot \boldsymbol{u} + \nabla^2 \Theta \right), && (1) \\
\frac{\partial \xi}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\xi &= \frac{1}{Sc} \left(2\, \boldsymbol{r} \cdot \boldsymbol{u} + \nabla^2 \xi \right), && (2) \\
\boldsymbol{\nabla} \cdot \boldsymbol{u} &= 0, && (3)
\end{aligned}
$$

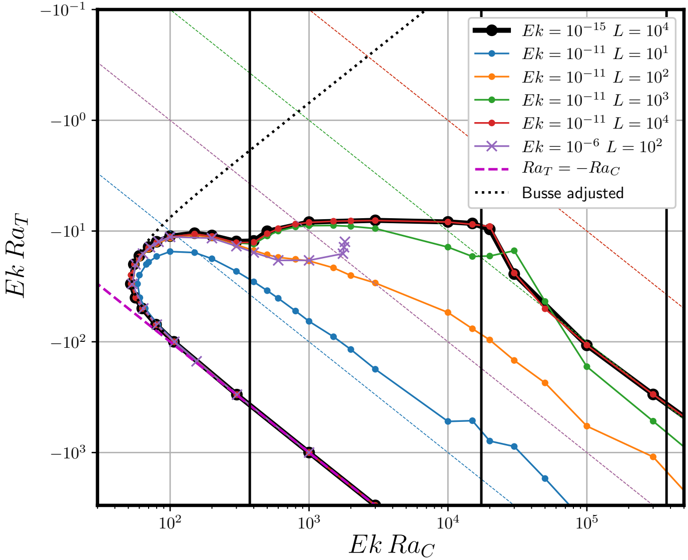
Numerical onset with SINGE: global survey

- Almost no effect except for $Ra_T \sim -Ra_C$.
- Onset in Ra_C is decreased by a *stabilizing* thermal gradient!
- $m = 1$ becomes most unstable mode.
- At constant Ra_T, the onset is not uniquely defined anymore.

All this has also been observed in previous studies (Simitev 2010, Net+ 2012).

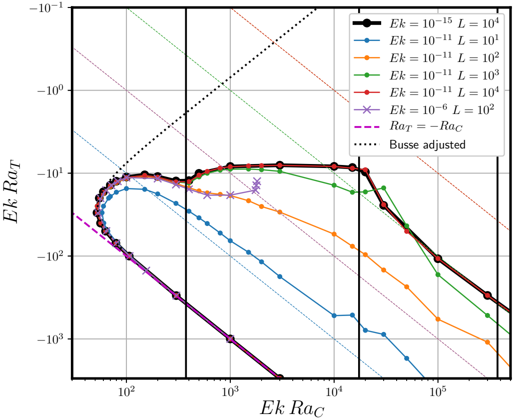


The anomalous $m = 1$ mode: INVISCID convection



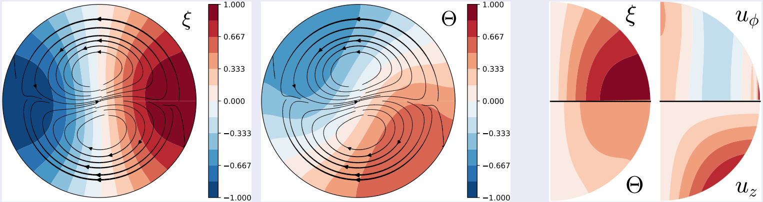
The anomalous $m = 1$ mode: INVISCID convection

- Inviscid onset evolves like $Ra \sim E^{-1}$.
- Viscous onset (e.g. $Ra_T = 0$) evolves like $Ra \sim E^{-4/3}$.
- Drop due to double-diffusive effect: $\sim E^{-1/3}$.

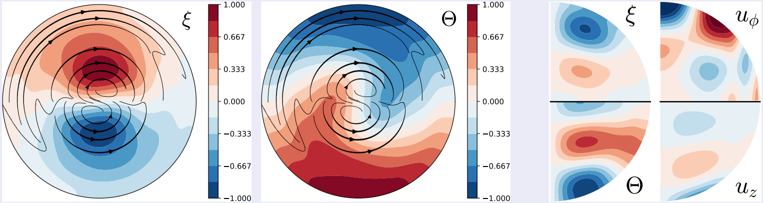


What does it look like?

$m = 1,\ Ek\,Ra_T = -25,\ Ek\,Ra_C = 52.5,\ Le = 1000$

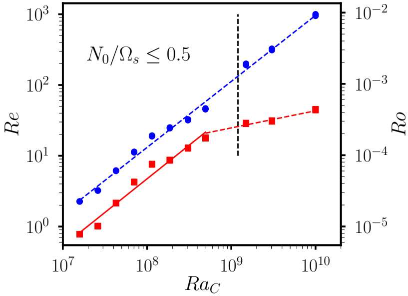


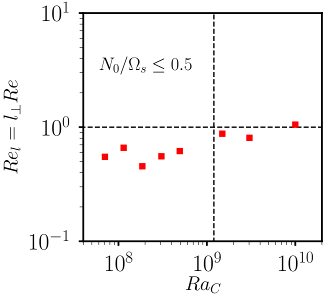
$m = 1,\ Ek\,Ra_T = -9,\ Ek\,Ra_C = 3000,\ Le = 1000$



Non-linear study (XSHELLS)

With $Ra_C = -3Ra_T$ (stably-stratified), $Le = 10$, $E = 10^{-5}$.





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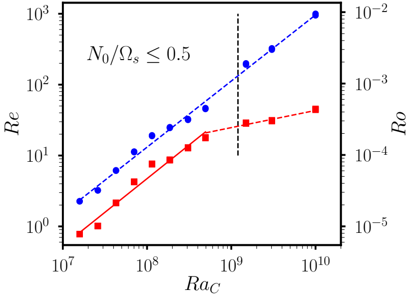

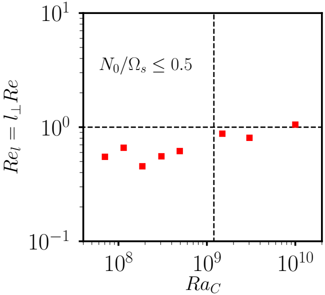

More details: Monville+ (GJI 2019) arXiv:1902.08523
code: `https://nschaeff.bitbucket.io/xshells/`

Conclusion for double-diffusive convection

Rotating double-diffusive convection in the finger regime:

Linear onset

- We have evidenced a large-scale ($m = 1$), invisicd convection mode due to combined effect of rotation, double-diffusion, and slope (β-effect).
- For Earth's core, we can compute the onset with SINGE; the onset is reduced by a factor 10^4.

Non-linear regime

- Not quasi-geostrophic.
- Large global Reynolds number Re.
- Production of strong zonal flows.
- Probably too weak radial flow for efficient dynamo generation.
- Typical size and flow velocity seem to adjust so that $Re_{loc} \sim 1$.

Monville+ (GJI 2019) arXiv:1902.08523

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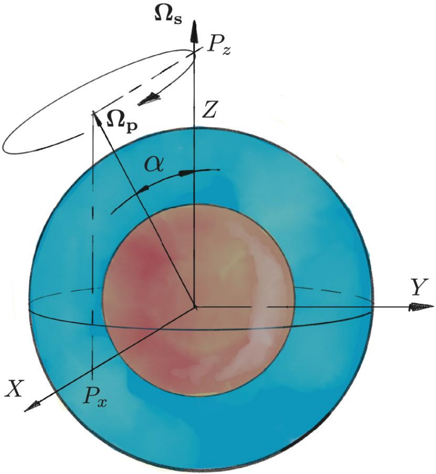
Motivation: better understand precession in fluid cores

- Precession can drive turbulent flows (experiments and numerics)
- Convection in planetary cores is not always enough to drive a dynamo (e.g. Moon)
- How much turbulence/mixing can we get from precession?
- How much dissipation? (important for orbital evolution, ...)

Precession forcing



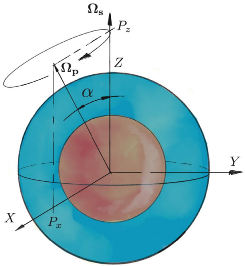
Basic solution: the fluid spins around yet another axis !

Method: direct numerical simulations in spheres

To simulate precession-driven dynamos in full spheres and spherical shells, we use the XSHELLS code `https://bitbucket.org/nschaeff/xshells`.

We ran 900+ simulations, among which 173 dynamos.

We solve the MHD equations in the precessing frame (where we see the planet spinning at Ω). This frame rotates at rate $Po\,\Omega$ along an axis $\hat{\mathrm{p}}$ making an angle α with the spin axis $\hat{\mathrm{z}}$.



$$\frac{\partial \mathrm{u}}{\partial t} + \mathrm{u}\cdot\nabla\mathrm{u} = -\nabla p + E\,\nabla^2\mathrm{u} - 2Po\,\hat{\mathrm{p}}\times\mathrm{u} + (\nabla\times\mathrm{B})\times\mathrm{B}, \qquad (4)$$

$$\frac{\partial \mathrm{B}}{\partial t} = \frac{E}{Pm}\,\nabla^2\mathrm{B} + \nabla\times(\mathrm{u}\times\mathrm{B}) \qquad (5)$$

Why consider a sphere?

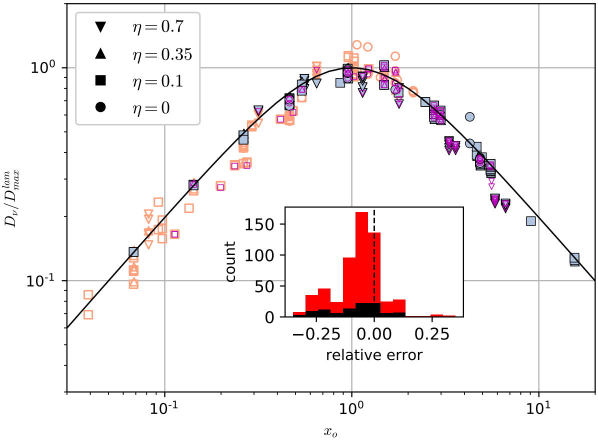
Pros:

- Efficient numerical methods ⇒ more realistic simulations.
- Some effects (shearing instabilities) are relevant.
- Need to understand the sphere, to have something to compare non-sphere with.

Cons:

- Rely on viscous coupling only (expected to be inefficient in planetary cores)

Viscous dissipation



$$D_\nu = \frac{2x_o}{1 + x_o^2} D_{max}$$

with:

$x_o \propto E^{1/2} / Po$

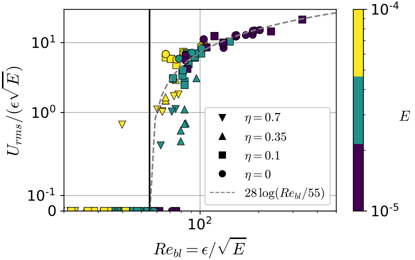
filled = stable;
magenta = MHD.

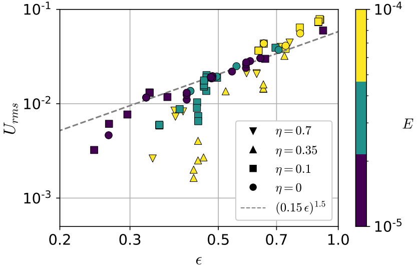
Viscous dissipation is dominated by friction of the solid-body rotating fluid on the solid outer sphere.

Turbulence level reached by precession forcing?

Key parameter: differential rotation $0 \leq \epsilon \leq 1$ between fluid core and mantle.

Two ways to collapse the turbulent fluctuation velocity U_{rms} of the 84 simulations with $Po < 0.1$, $E \leq 10^{-4}$ and low magnetic energy.





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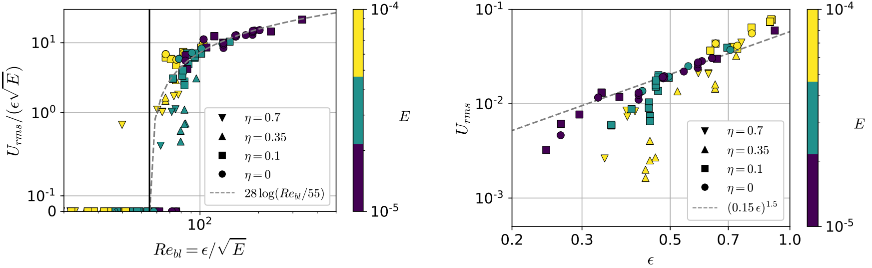

- low amplitude fluctuations $\epsilon\sqrt{E} \lesssim U_{rms} < \epsilon/10$ in the planetary parameter range.

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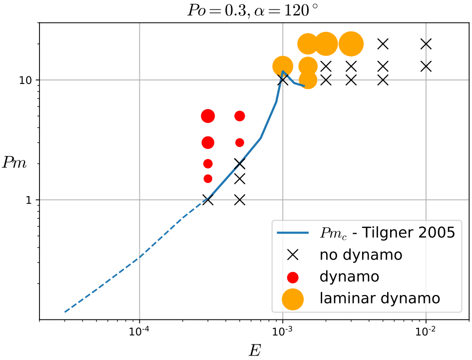
Precession dynamos in a sphere, following Tilgner 2005

- *Pm* $= \nu\, \mu_0 \sigma$ (non-dimensional measure of electrical conductivity σ; for liquid metals *Pm* $\sim 10^{-5}$.
- Ekman number *E* compares viscosity and planet spin (moon *E* $\sim 10^{-11}$).



- We reproduce the results of Tilgner 2005

Precession dynamos in a sphere, following Tilgner 2005

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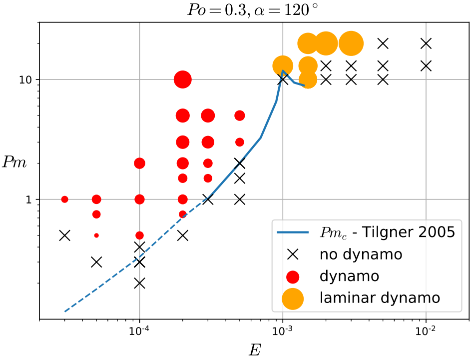

- We reproduce the results of Tilgner 2005
- We lower the viscosity

Dynamo action does not look good at low viscosity...

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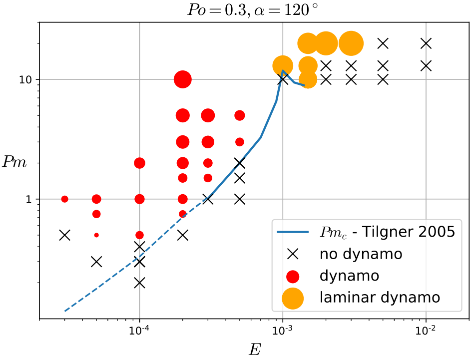

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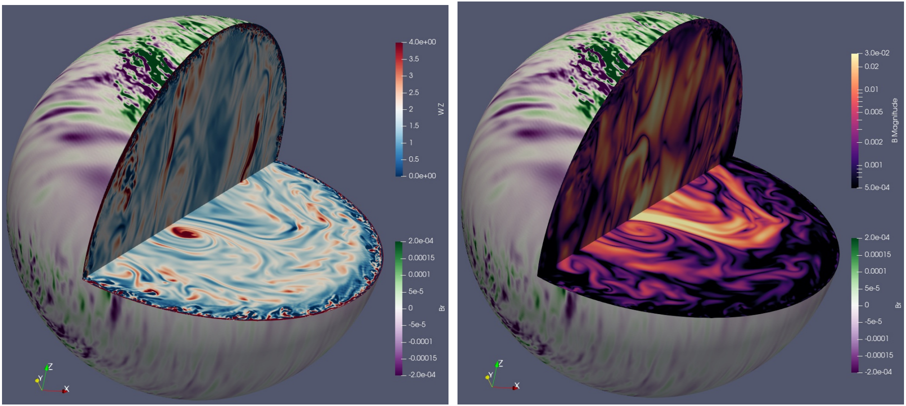
Dynamo action does not look good at low viscosity...

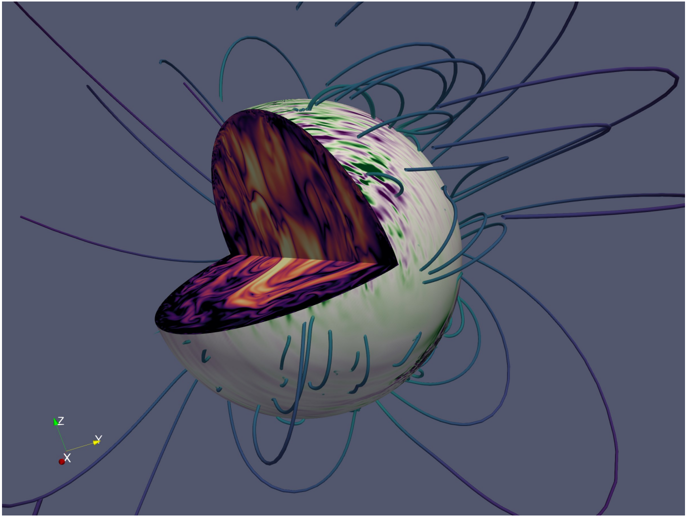
Same thing in the cube (Goepfert&Tilgner 2016,2018)

Precession dynamo snapshot



Snapshots (corotating with the fluid) at $E = 10^{-5}$, $Pm = 0.3$, $Po = 0.007$, $\alpha = 90°$.

Small-scale, multipolar field generated



This will look dipolar at the moon surface, but most of the magnetic flux will not make it to the surface... (weak field at the surface)

Conclusions for precession in spheres

Precession-driven flows at low viscosity:

- turublence level reached in planetary cores is poorly constrained.

Precession-driven dynamos at low viscosity:

- sparsely populate the parameter space.
- **Dissipation power cannot be used for scaling the magnetic field.**
- For planetary cores: flow and field peak at small scales...

Possible Workarounds:

- Topographic coupling (deviation from sphere)? (e.g. Reddy+2018)
- Combined action of precession and convection? (e.g. Wei 2016)
- Convective dynamo generated by the viscous heating (Stys & Dumberry 2020)

Paper : *Cebron+* GJI 2019, arXiv:1809.05330.

Simulation database available, doi:10.6084/m9.figshare.7017137

Overall conclusions

- Precession in a sphere may dissipate a lot of power, but does not easily lead to a dynamo.
- Large-scale flows easily penetrate a stably stratified layer.
- Waves can be trapped near the equator by a stably stratified layer.
- Tides can erode a stratification if $N < \Omega$.
- Double-diffusive effects can lead to large-scale, inviscid convective motions in a stably-stratified layer, including strong zonal flows.